Prioritised Unit Propagation by Partitioning the Watch Lists

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Pragmatics of SAT
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1) Introduction
2) Priority Propagation
3) Heuristics for Prioritisation
4) Experimental Results
5) Conclusion

Introduction

Boolean Satisfiability Problem (SAT)

- **Boolean Formula** given in **Conjunctive Normal Form**:

$$F = \bigwedge_{i=1}^{m} \left(\bigvee_{j=1}^{k_i} l_{ij} \right) = ((l_{11} \vee \ldots \vee l_{1k_1}) \wedge \ldots \wedge (l_{m1} \vee \ldots \vee l_{mk_m}))$$

- Disjunctions $\bigvee_{j=1}^{k_i} l_{ij}$ are called **clauses**

- Clauses consist of **literals** $l_{ij} \in \{x_1, \neg x_1, x_2, \neg x_2, \ldots, x_n, \neg x_n\}$

  for variables $x_1, x_2, \ldots, x_n$

- Variables can be assigned $\{\textbf{true}, \textbf{false}\}$

SAT Solving

- to solve a satisfiable problem **(SAT)**:
  - Assign variables with each clause having a true literal
  - **NP-hard**

- to solve an unsatisfiable problem **(UNSAT)**:
  - proof that no such assignment exists

State-of-the-Art SAT Solving

- Modern SAT Solvers use

  Conflict Driven Clause Learning (CDCL)

- Basic Idea:
  - Choose a partial assignment of the variables
  - For all clauses check for conflicts and implications
  - Analyse conflicts and learn conflict clauses

Search for Conflicts & Implications

- Accomplished by **(Unit) Propagation**

- basic Idea:
  - all literals in a clause assigned **false** ==> Conflict
  - all but one literal assigned **false** and remaining literal unassigned ==> (**Unit) Implication**

Unit Propagation

- **traditional approaches** for fast unit propagation:
  - **deleting clauses** (inevitable)
  - **avoiding clause look-ups**
    - special data structures
      - e.g. **Two Watched Literals (TWL) scheme**

Priority Propagation (PriPro)

- Our contribution:

  adding prioritisation to unit propagation

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- Our Solver CaDiCaL_PriPro:

  Special Innovation Award at SAT Comp. 2021

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- **basic idea**:
  - select clauses to prioritise
  - only propagate prioritised clauses
  - if no conflict occurred, propagate other clauses, too

Priority Propagation (PriPro)

- Our contribution:

  adding prioritisation to unit propagation

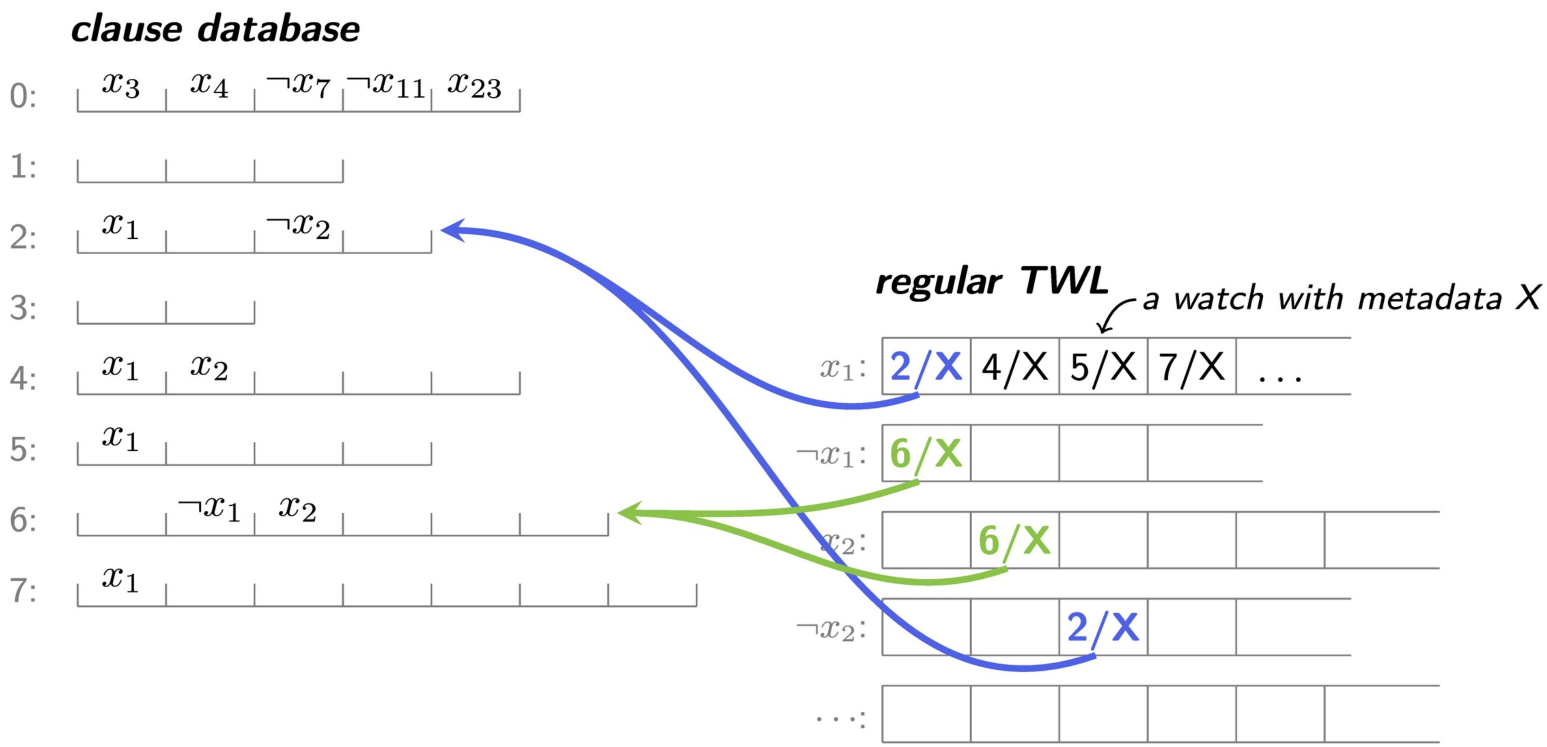
- **basic idea**:
  - select clauses to prioritise
  - only propagate prioritised clauses
  - if no conflict occurred, propagate other clauses, too

- implementation:
  - **additional TWL scheme** for prioritised clauses
  - **adapt unit propagation** to focus on prioritised clauses

Priority Propagation (PriPro)

Two Watched Literals

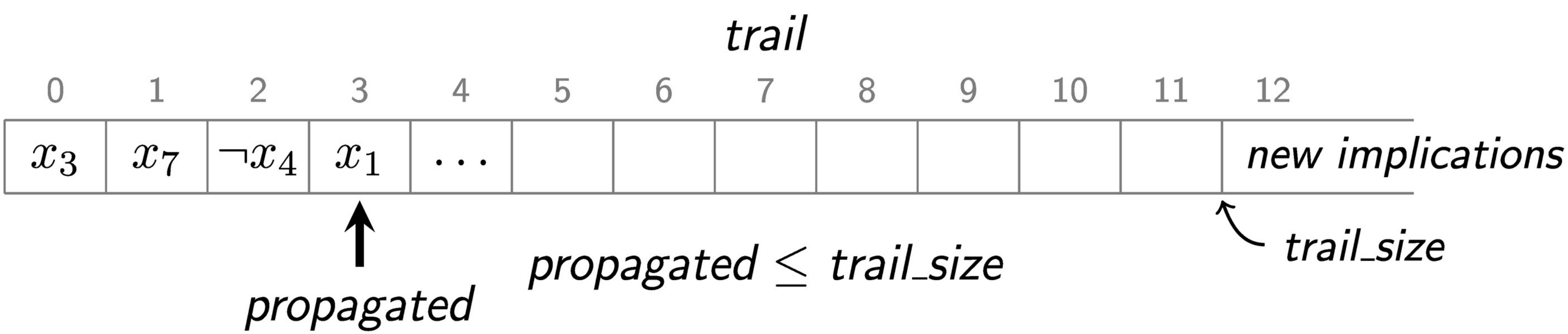
Two Watched Literals



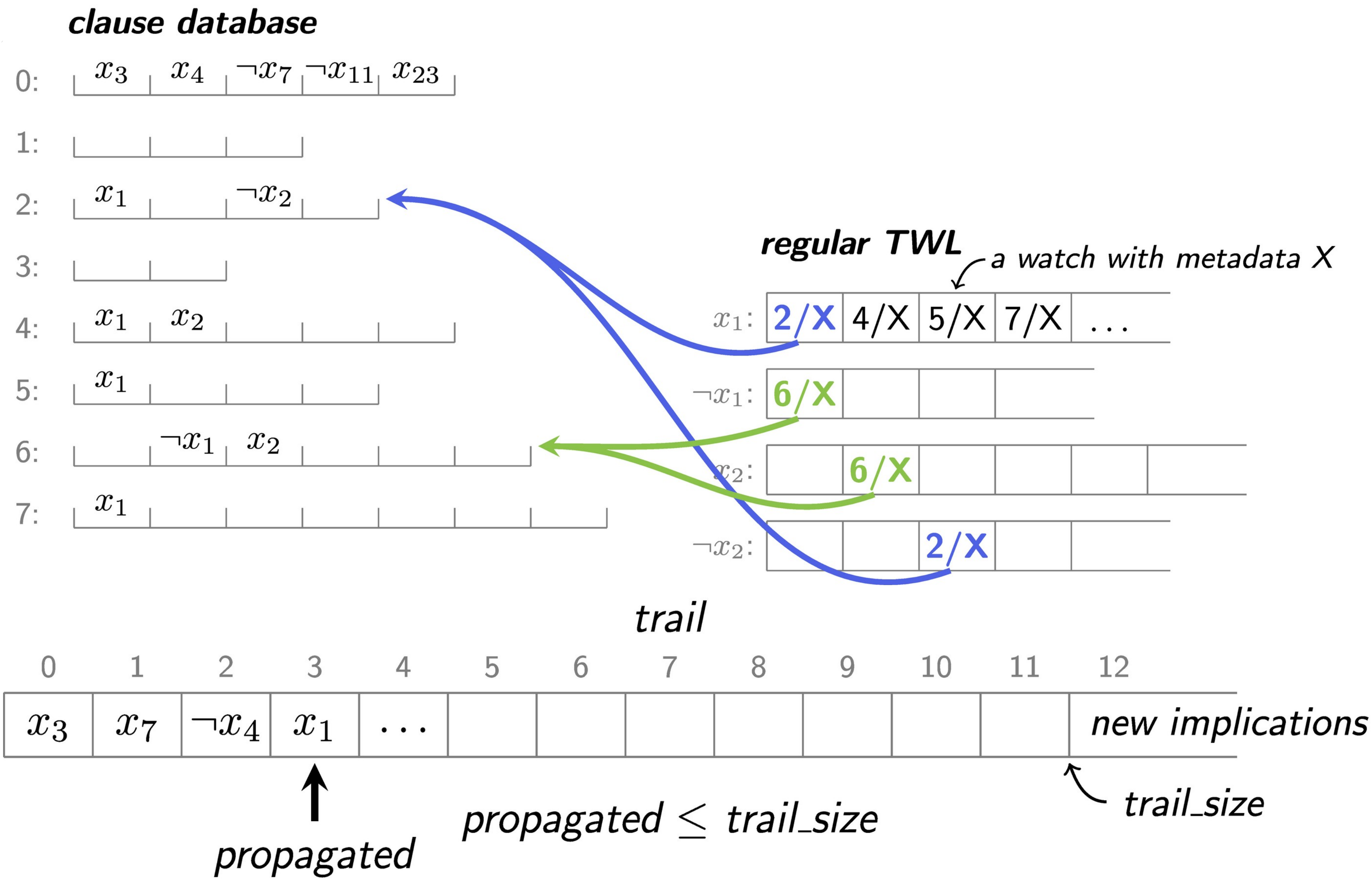
- basic idea:
  - only look clauses up if one of their watched literals is **false**
  - only those clauses can lead to conflict or implication

Trail

- **trail** indicates which literals have been assigned **true**
- contains literals **in order of assignment**
- (negations of) literals on the trail need to be **‘propagated’**
- **iterate through trail** to propagate literals



Propagation of Literals



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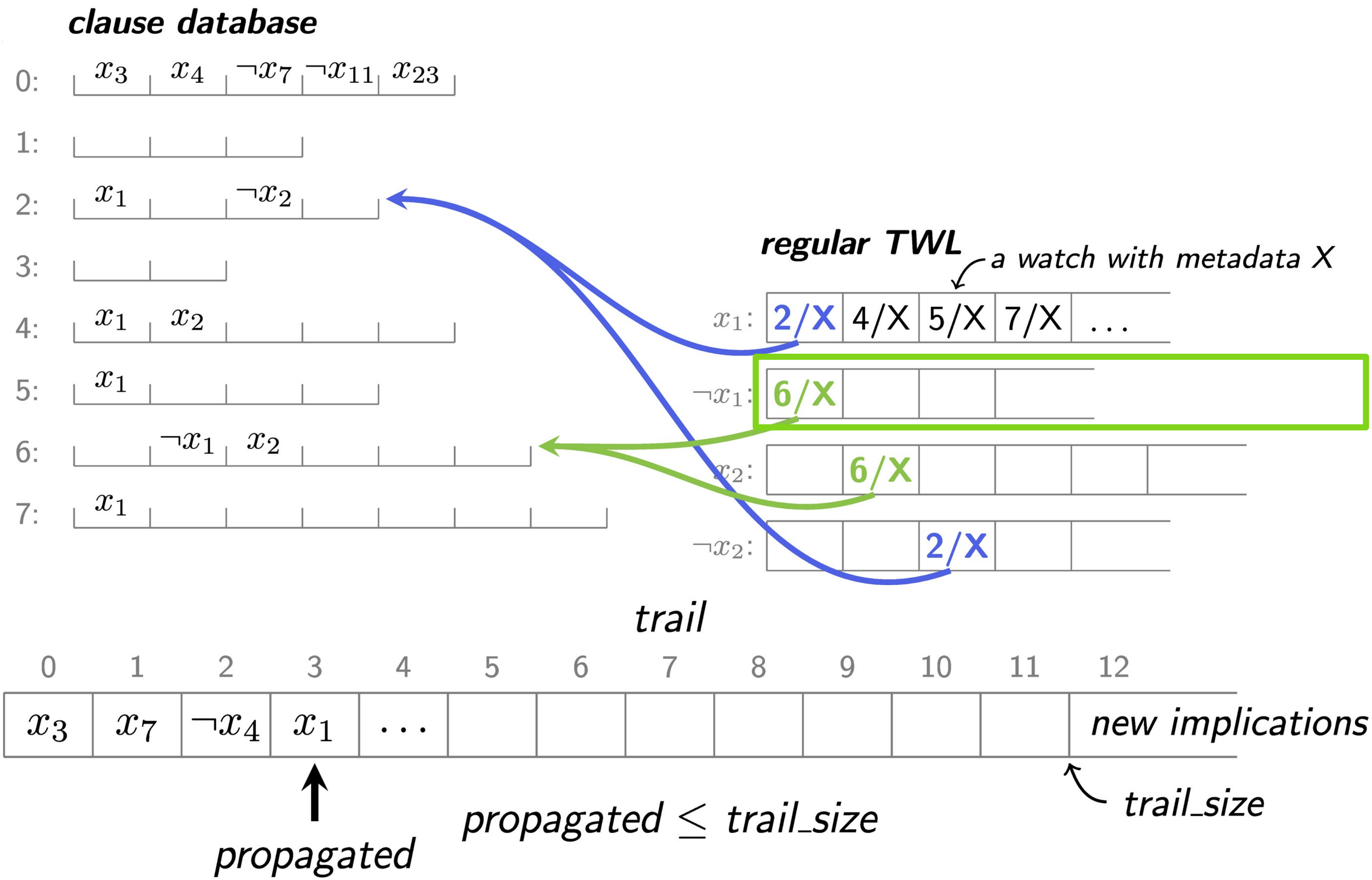

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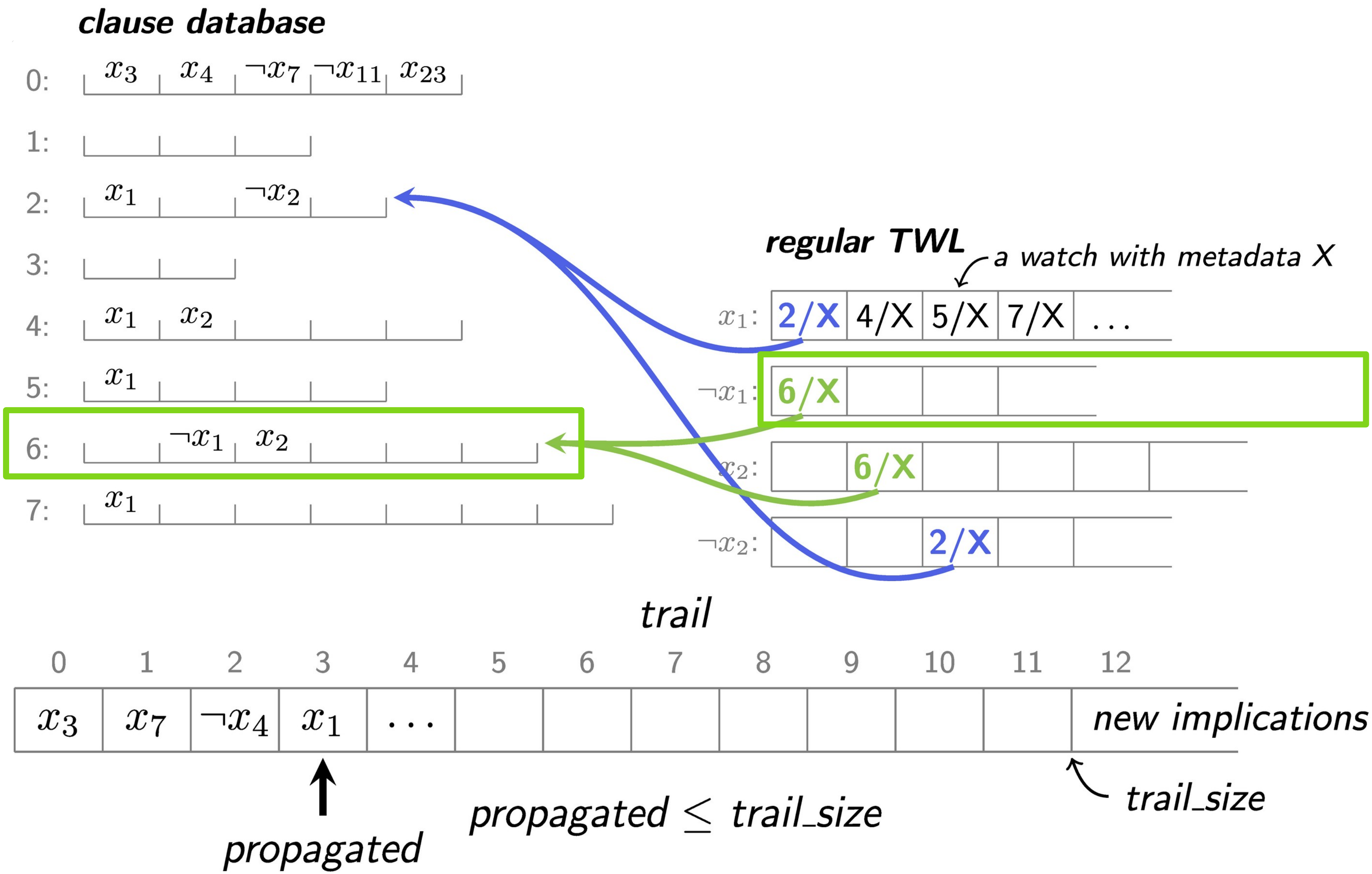
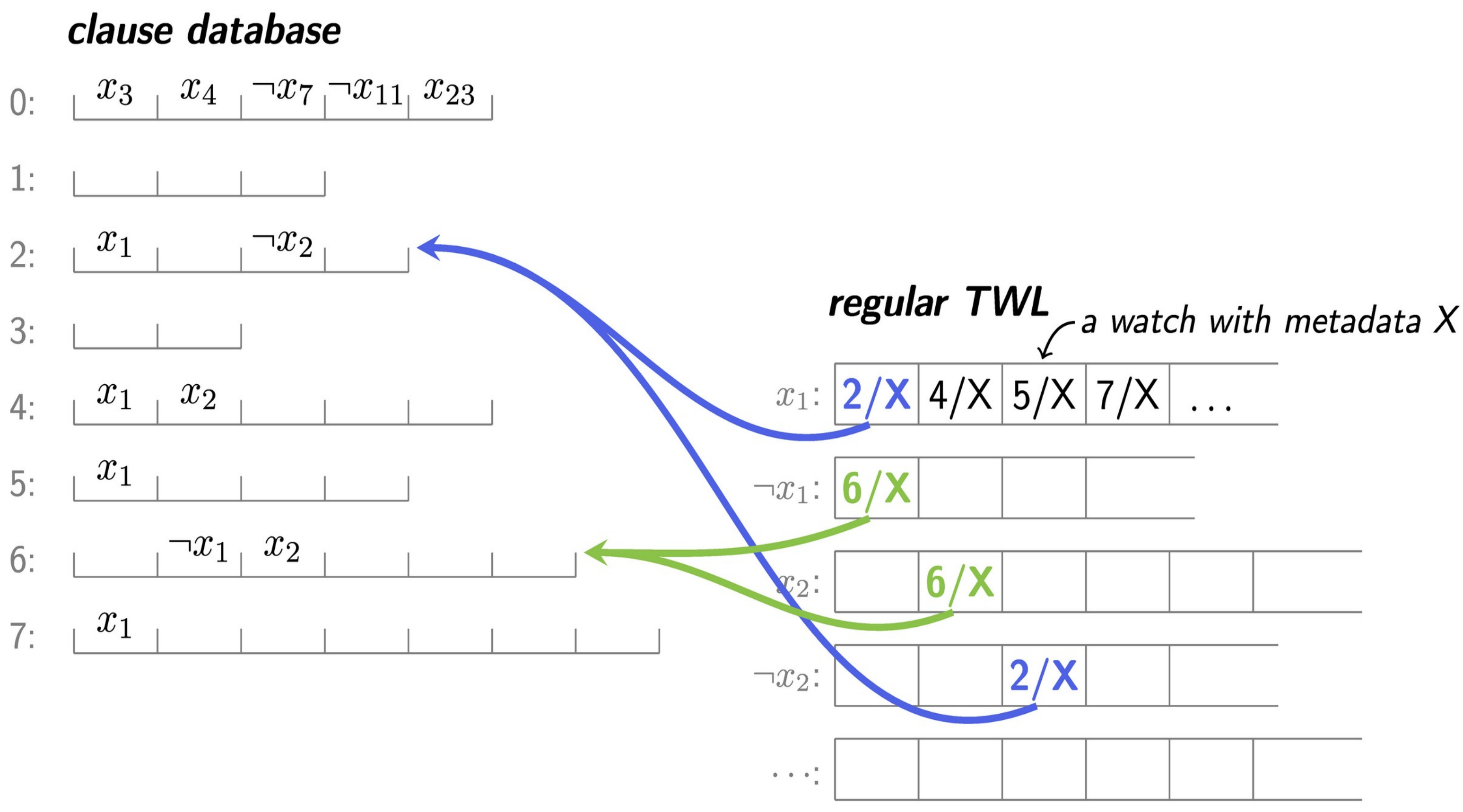


Prioritised TWL scheme



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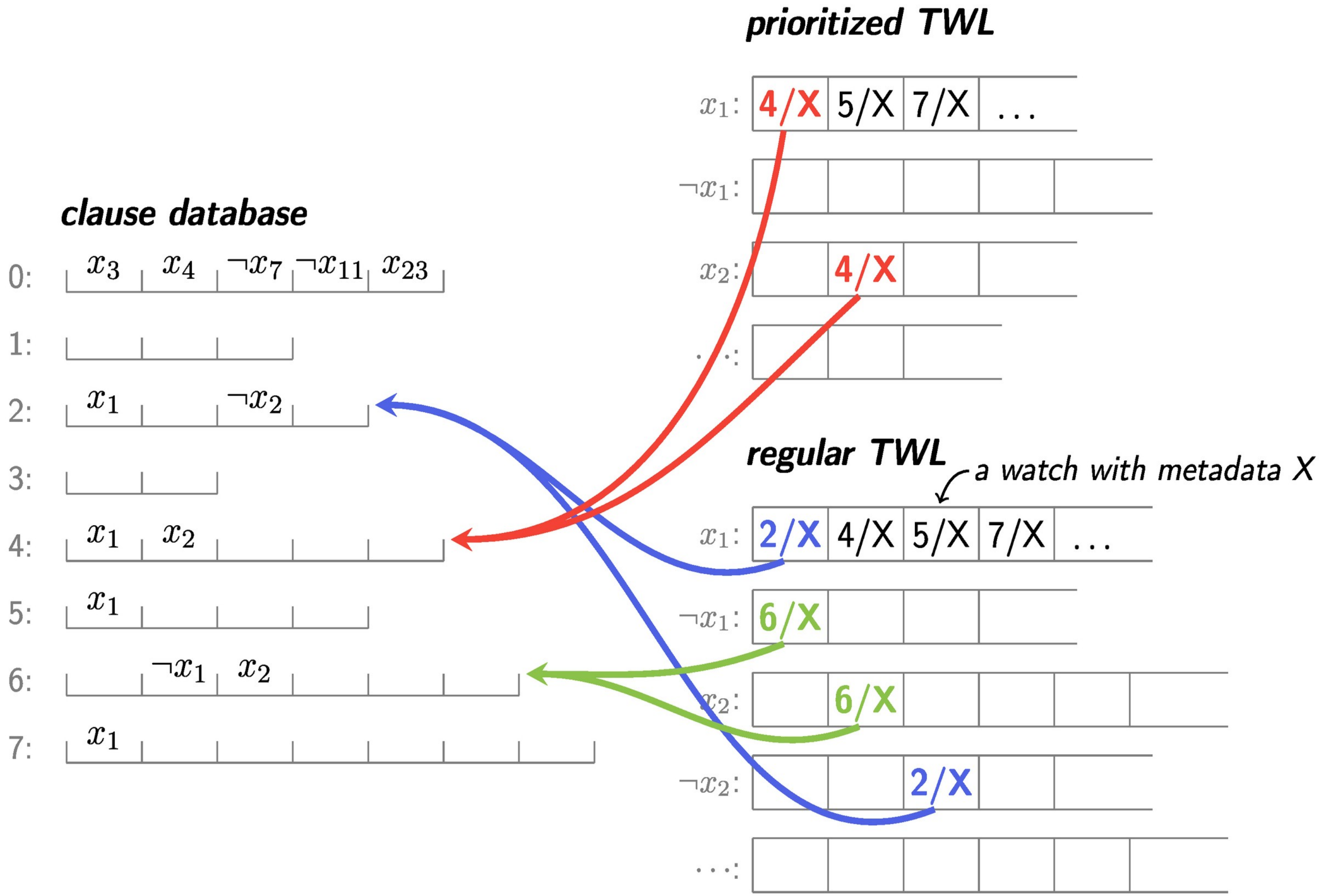

Unit Propagation with PriPro

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Algorithm Unit Propagation

while $\neg conflict \wedge propagated \neq trail\_size$ **do**

  $lit \leftarrow trail[propagated]$

  $propagated \leftarrow propagated + 1$

  $conflict \leftarrow propagate\_watches\_of\ (\neg lit)$

end while

return $conflict$

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  end if

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of **all** remaining literals

Propagation of Prioritised Watches

- resembles original unit propagation

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- distinguish between
  - current literal w.r.t. regular watches
    - indicated by **propagated**
  - current literal w.r.t. prioritised watches
    - indicated by **pripro_propagated**

Propagation of Prioritised Watches

- resembles original unit propagation

Algorithm The *propagate_prioritised_watches* function

while $\neg conflict \land pripro\_propagated \neq trail\_size$ **do**

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Relation between Current Literals

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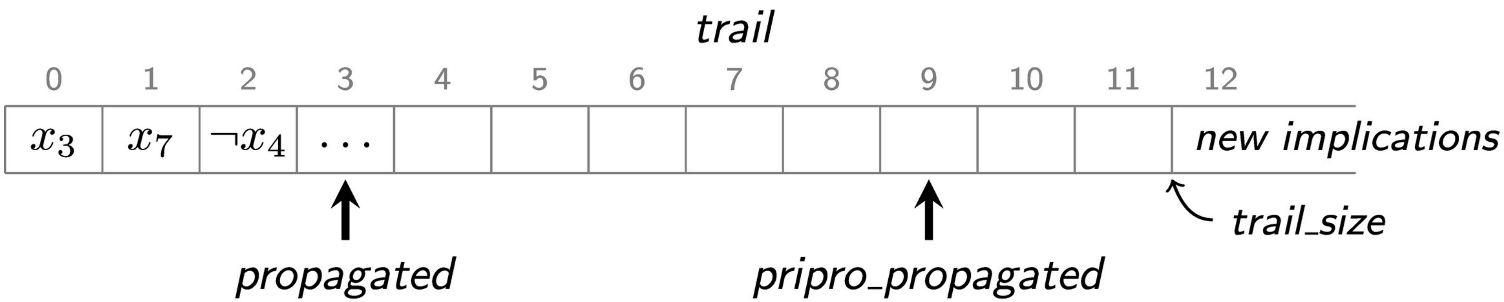

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Relation between Current Literals

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- current literal ≈ next to be propagated

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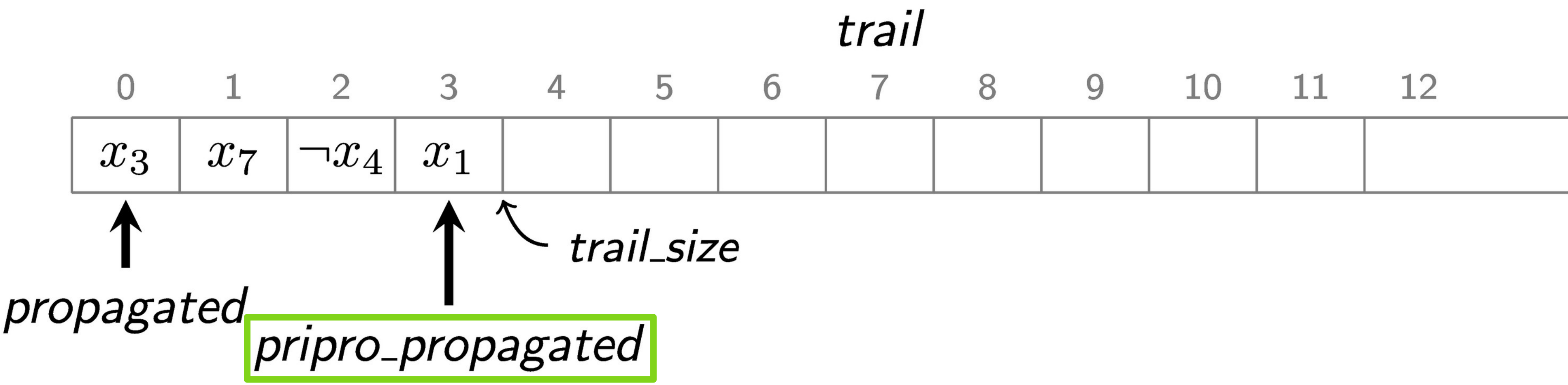

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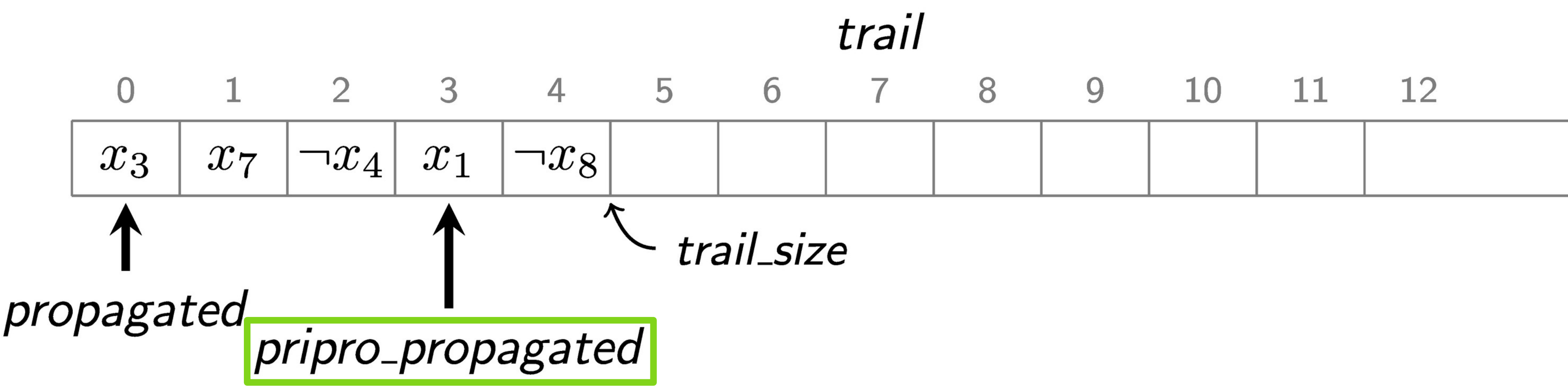

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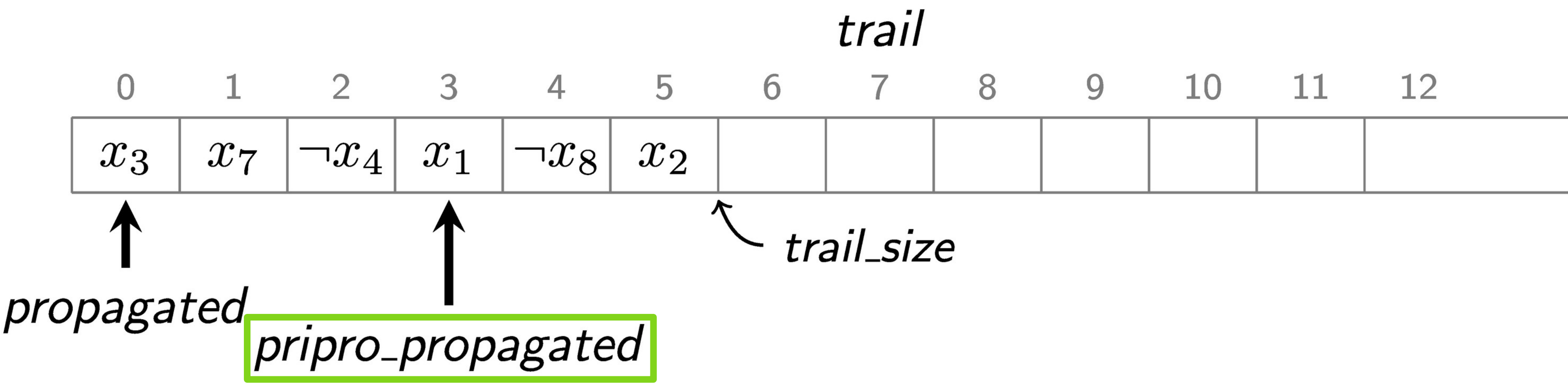

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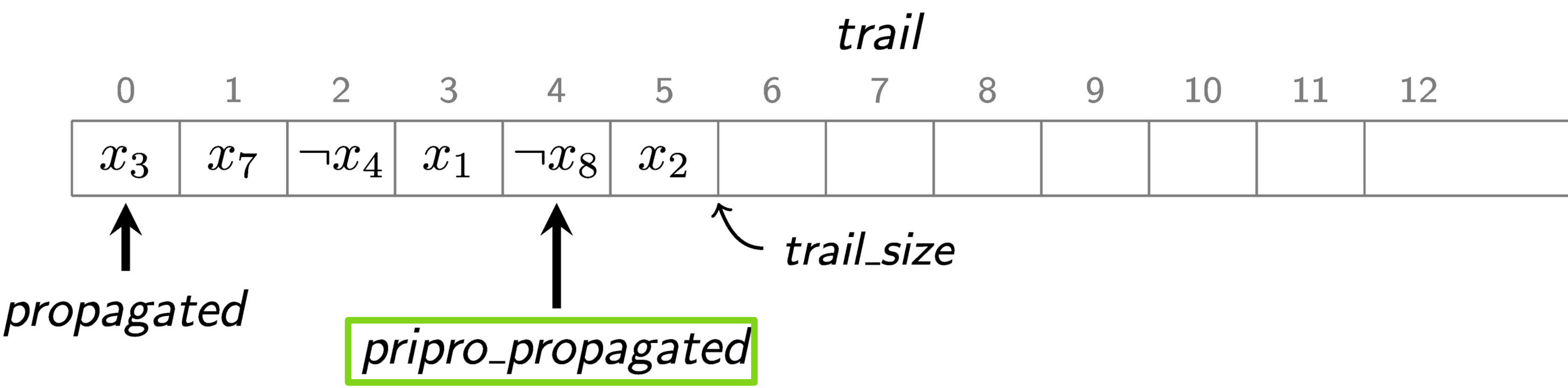

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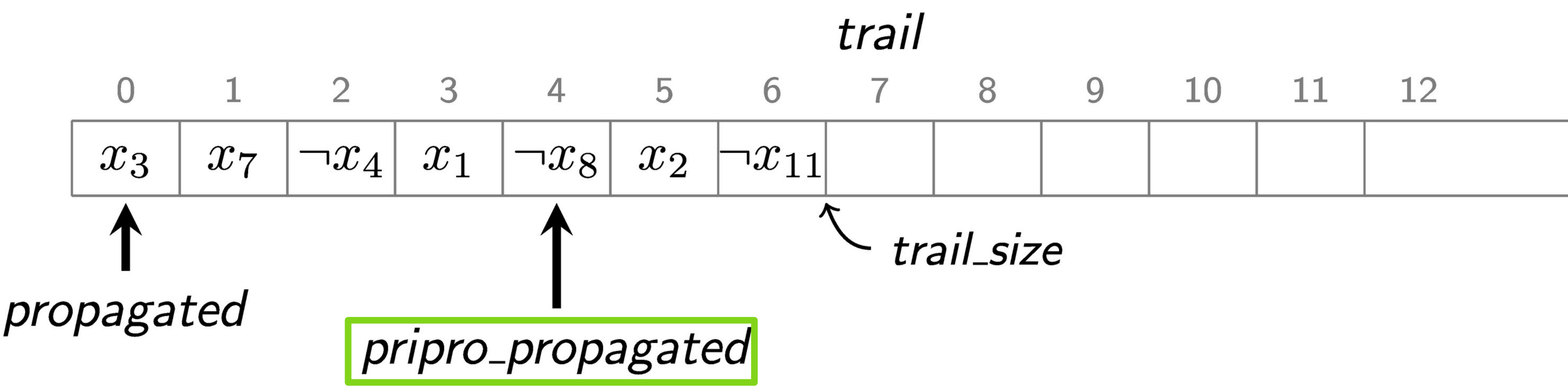

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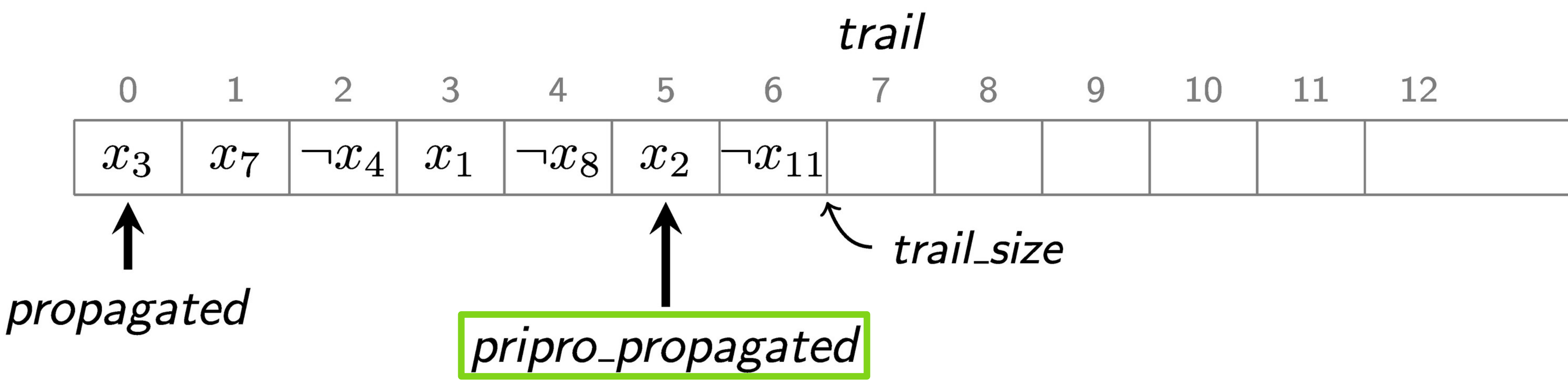

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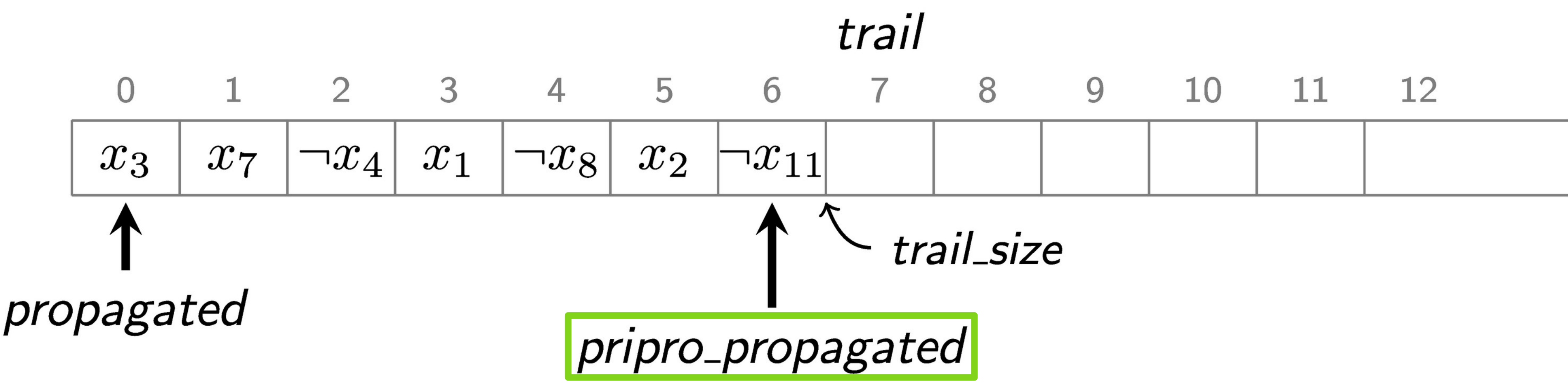

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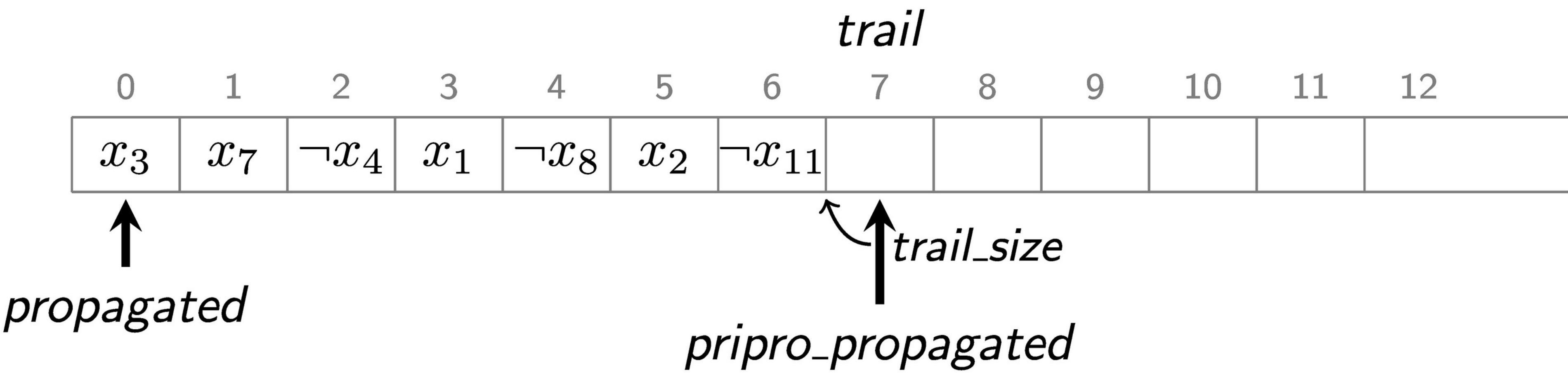

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Relation between Current Literals

trail

0	1	2	3	4	5	6	7	8	9	10	11	12
x_3	x_7	$\neg x_4$	x_1	$\neg x_8$	x_2	$\neg x_{11}$						

$\uparrow$ *propagated*

$\uparrow$ *trail_size*

pripro_propagated

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propagated

trail_size

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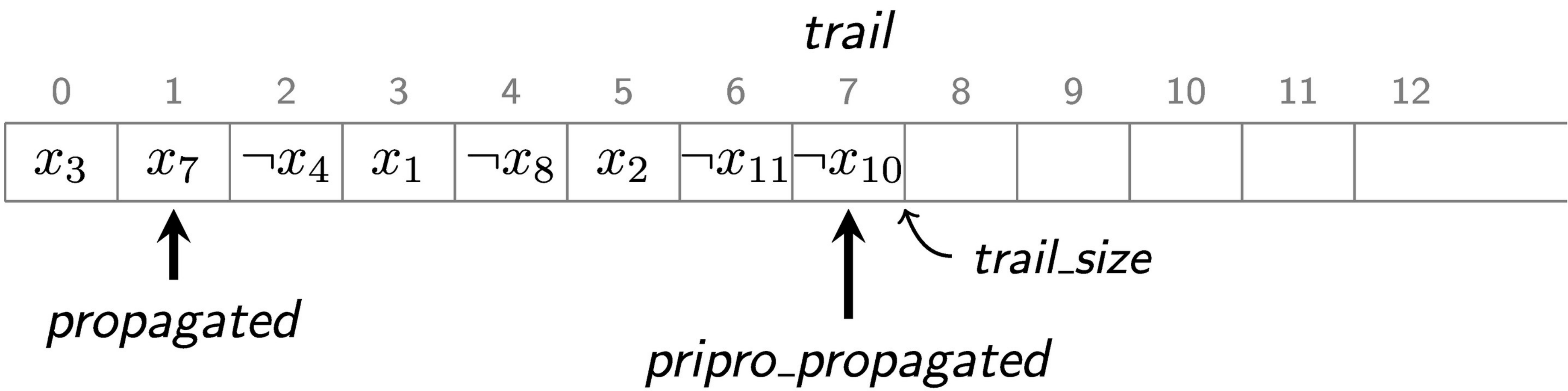

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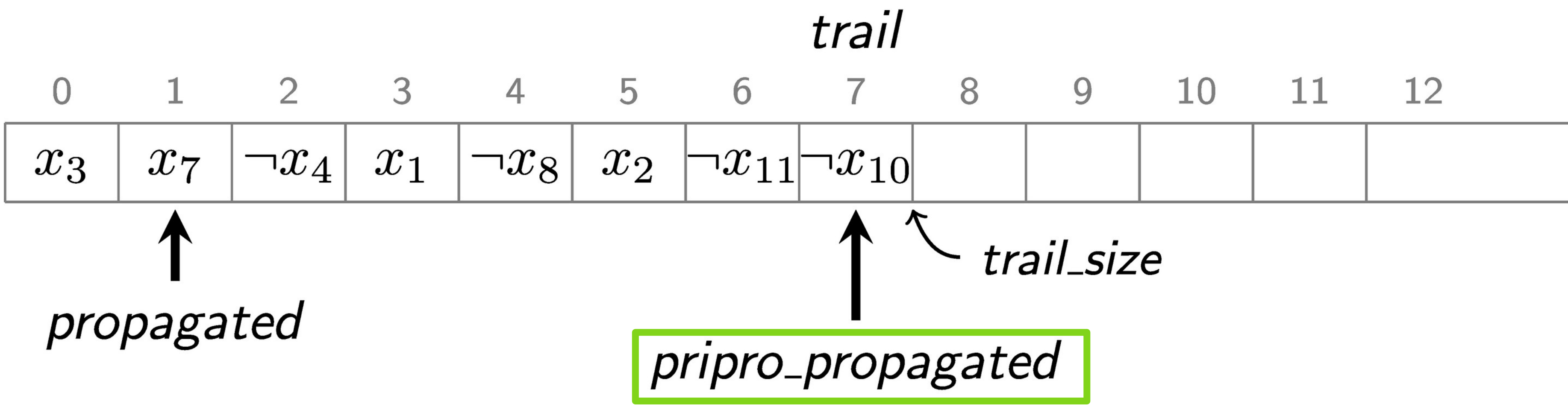

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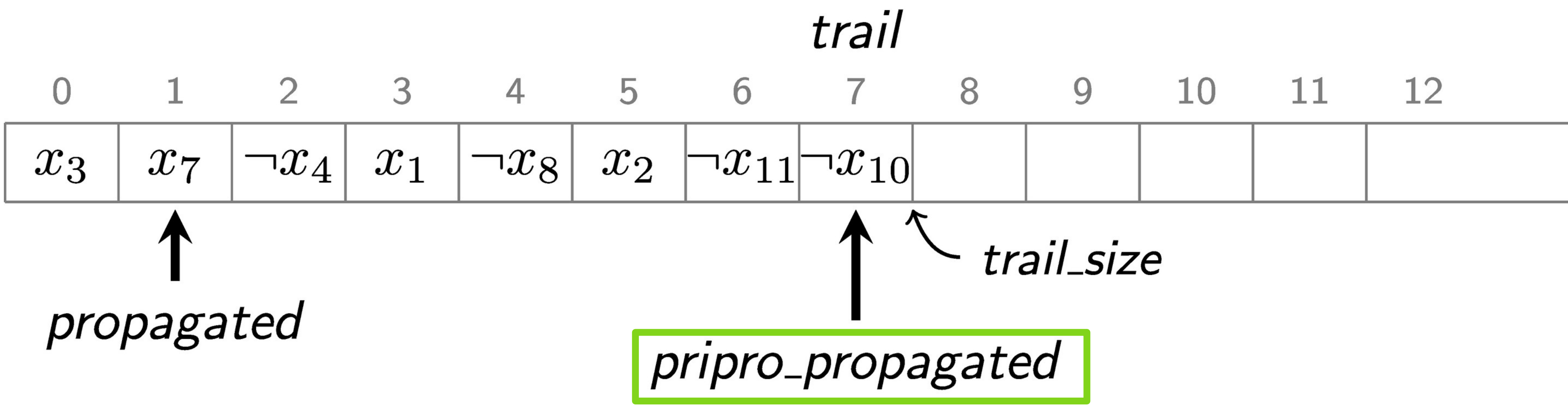

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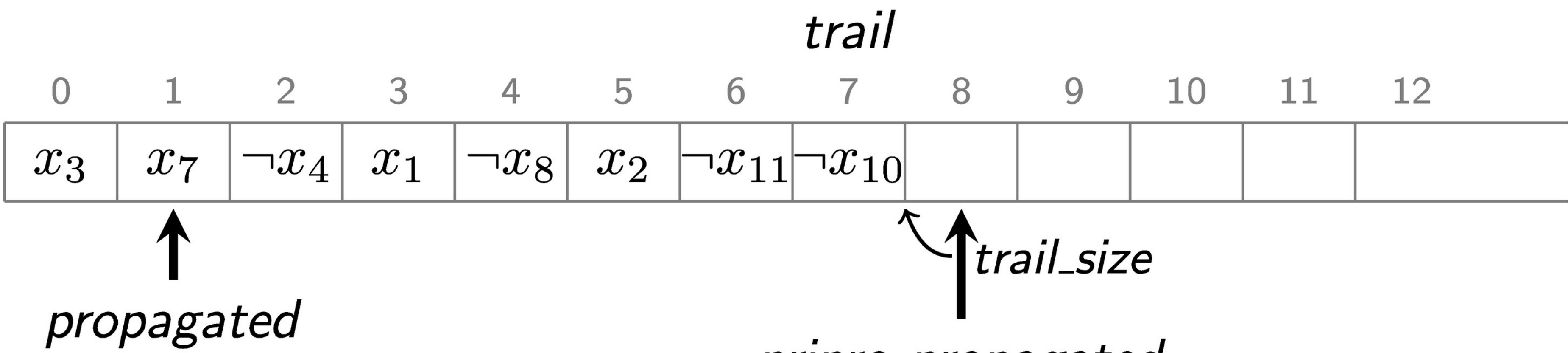

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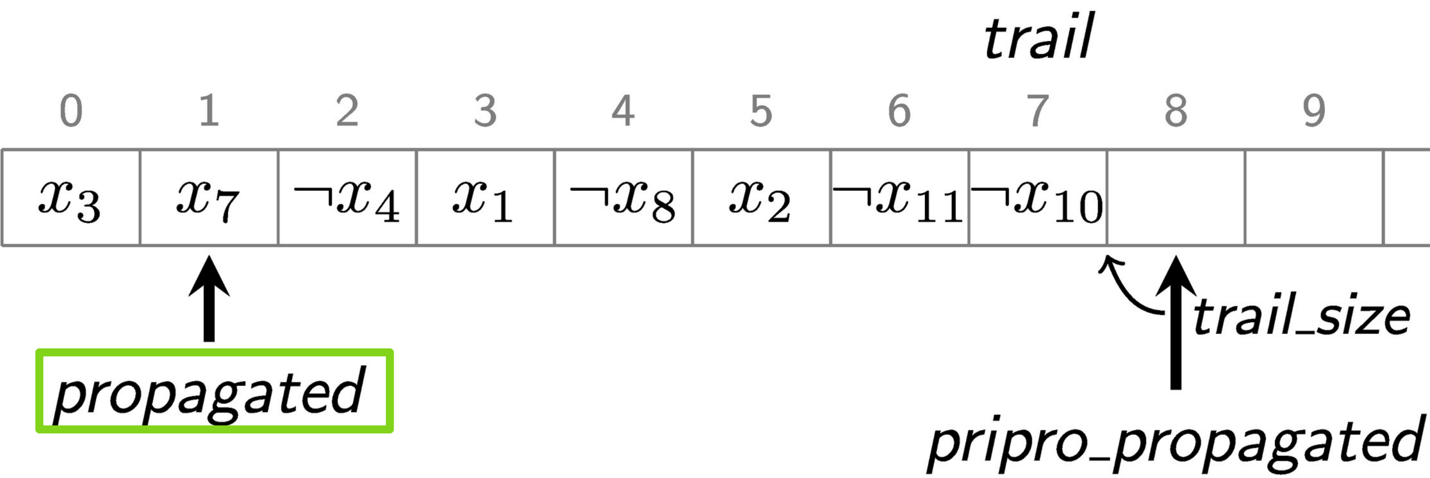

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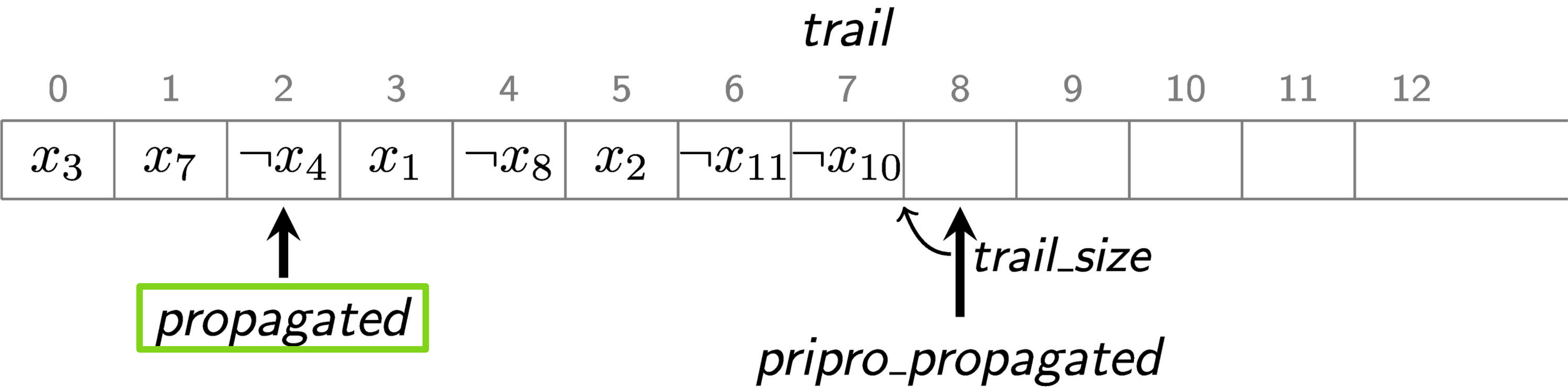

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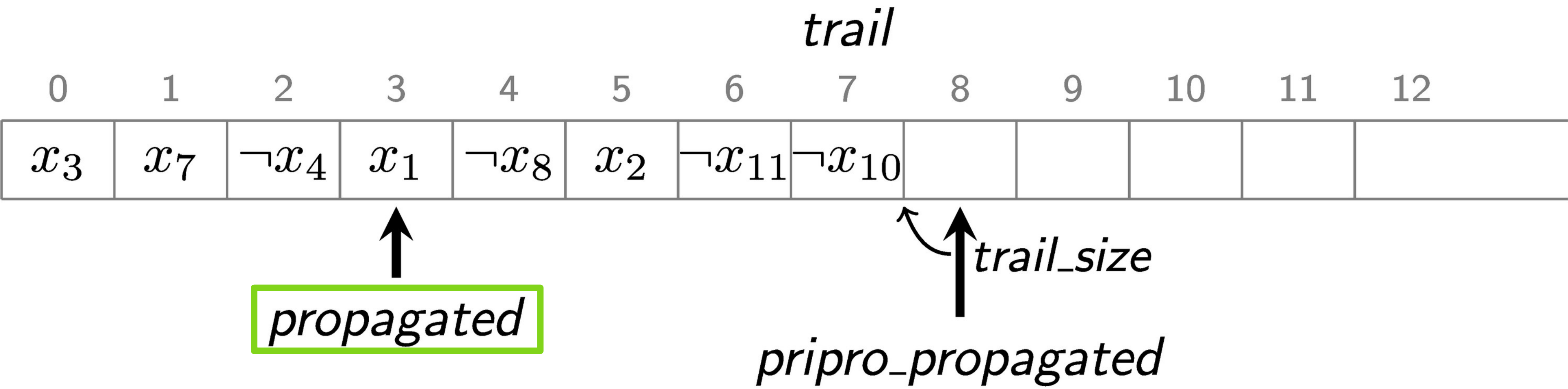

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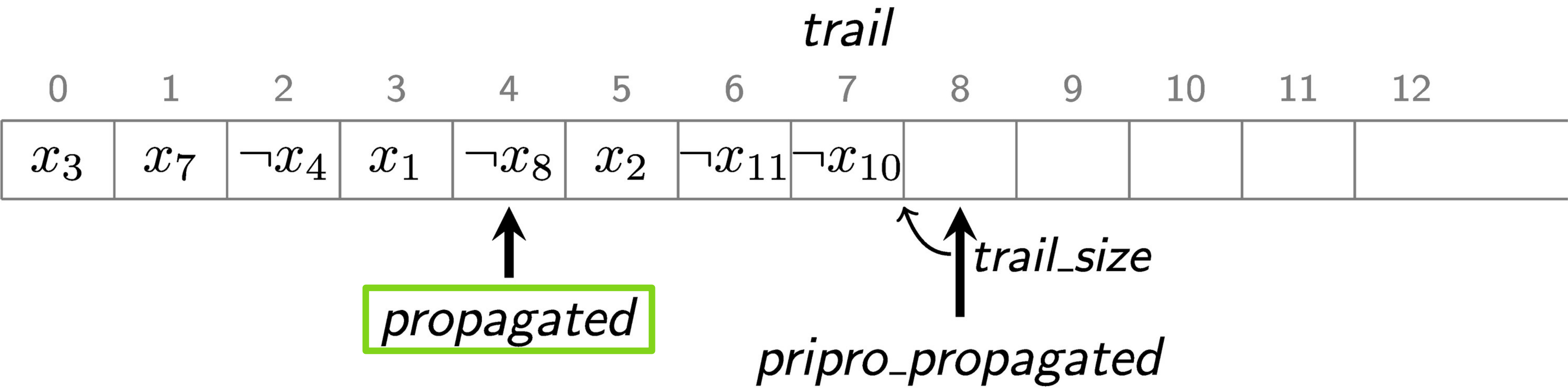

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trail_size

propagated

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Summary: Priority Propagation

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- considers the same clauses for propagation as before
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- propagation order differs
  - propagate all prioritised watches until end of trail
  - propagate regular watches of next literal on the trail
  - repeat until conflict or end of trail is reached

Heuristics for Prioritisation

Dynamic Prioritisation

- choose/adjust prioritised clauses at runtime

- to prioritise a clause (**upgrading)**:
  - move both watches:

    from regular to prioritised TWL scheme

- to de-prioritise a clause (**downgrading):**
  - move both watches:

    from prioritised to regular TWL scheme

Simple Heuristics

- parametrised upgrade heuristic
  - based on recent resolvents
  - upgrade on-the-fly

- parametrised downgrade heuristics
  - always downgrading all clauses at once
  - triggered by events + at constant interval

- other heuristics possible!

Upgrade Heuristic

- **upgrade** during conflict analysis**:**
  - clauses appearing in conflict analysis (**resolvents)**
    - only if size/LBD smaller than some threshold
    - **default:** LBD ≤ 6
    - (**conflicting clause** is never upgraded)
  - newly learned clauses (**conflict clauses**):
    - regardless of size/LBD

Downgrade Heuristic

- **sporadically downgrade all clauses at once**
  - **forced downgrades** triggered by
    - any inprocessing techniques (easy combination)
    - clause database reductions (i.e. clause deletions)
    - rephasing
    - optionally at restarting
  - **scheduled downgrades**:
    - at constant interval (default: 10.000 conflicts)
    - interval is measured in conflicts
    - regardless of whether forced downgrades occurred

Experimental Results

Experiments

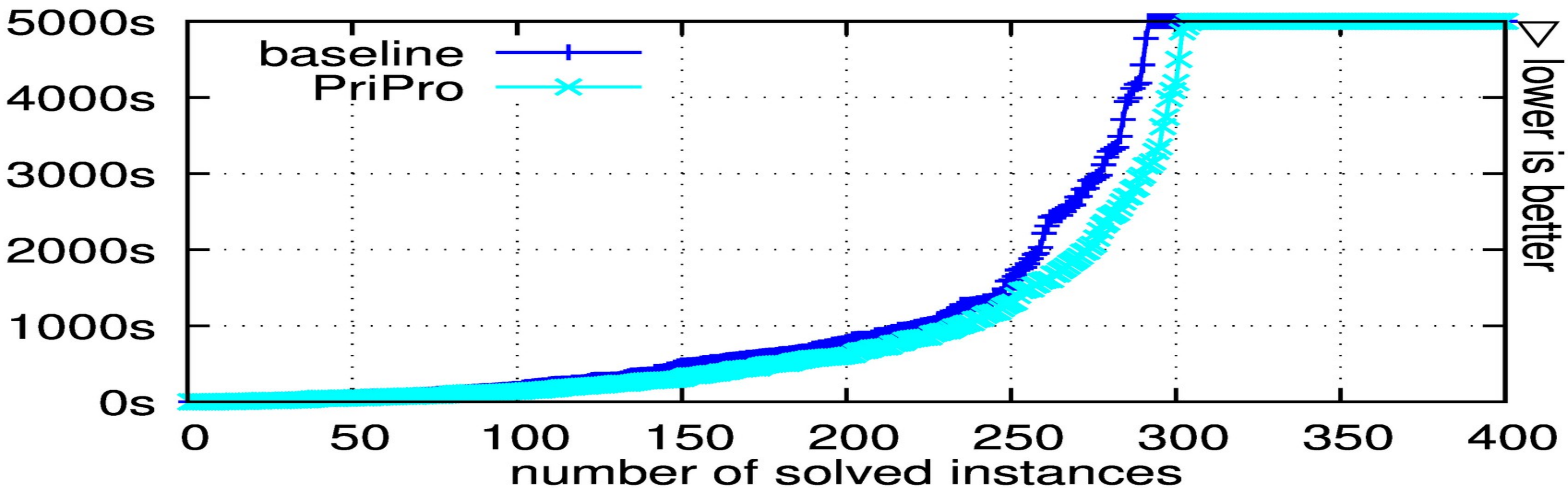
- tested on SAT Competition 2021 instances (400 problems)
- baseline solver: CaDiCaL v 1.4.0

- varying upgrade heuristics with
  - LBD limit: 0, 1, …, 8, no limit
  - size limit: 2, 4, …, 12, no limit

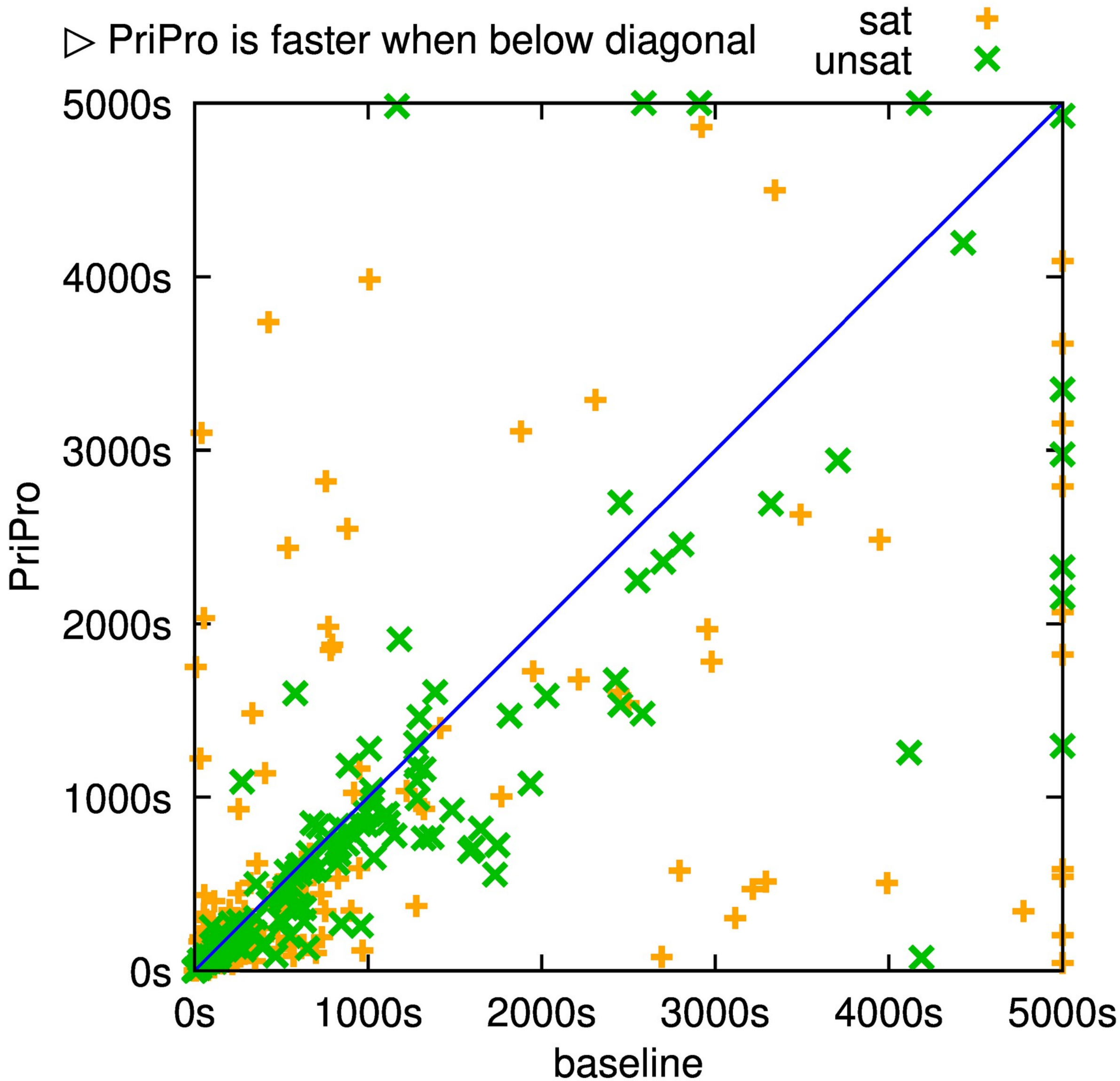
- downgrade heuristic
  - scheduled downgrade interval:
    2, 5, 10, 25, …, 1.000.000, only forced downgrades
  - optionally: downgrades at restarts

Good Results!

- **Improves performance** on both, SAT and UNSAT
  - mostly independent of parameters chosen
  - overall speed-up of about 10% (nPar2-score)
  - solved 5 to 15 instances more



Speed-up on UNSAT



Shorter Conflict Clauses Learned

- reduction of average size of learned clauses
- before learned clause minimisation:
  - about 7 % on SAT
  - about 11 % on UNSAT
- after learned clause minimisation:
  - about 6 % on SAT
  - about 21 % on UNSAT
- mostly independent of parameters chosen
- confirms hypothesis by Jingchao Chen [1]

[1] Jingchao Chen, Core First Unit Propagation, arXiv preprint: 1907.01192 (2019)

Conclusion

Summary: Priority Propagation

- **reordering clause look-ups** during Propagation
- **improves performance** on both, SAT and UNSAT
- reliable **speed-up** on UNSAT
- **smaller clauses learned** from conflict analysis
- already **simple heuristics** lead to good results
  - mostly **independent of parameters** chosen
- **easy to implement** in modern SAT solvers

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- **reorder clauses** for propagation
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Thank you for your attention.