

# Enhancing State-of-the-Art Parallel SAT Solvers Through Optimized Sharing Policies

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**Pragmatics of SAT 2023**



# Sequential enumeration reaches its limits

CDCL solvers are efficient thanks to:

- Preprocessing [[EB05](#), [PHS08](#)]
- Branching Heuristics [[ZMMM01](#), [LGPC16](#)]
- Qualifying learned clauses for garbage collection

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- Rarity of new heuristics
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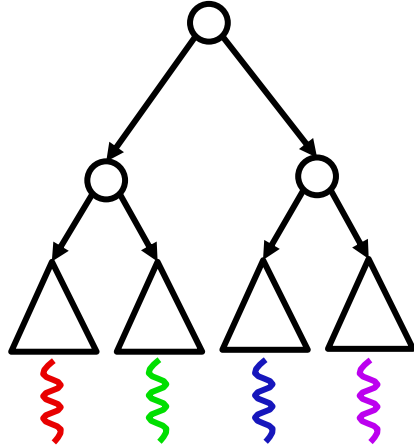
**Developing parallel SAT solvers able to exploit new multicore machines become a necessity.**

# Interest in parallel SAT solving

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**Divide and conquer**

[ZBH96]

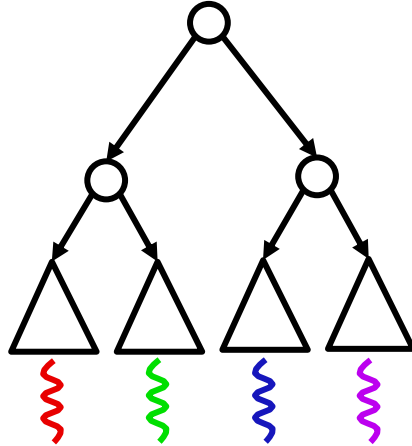


Dynamically divide the search space  
between each worker

# Interest in parallel SAT solving

## Divide and conquer

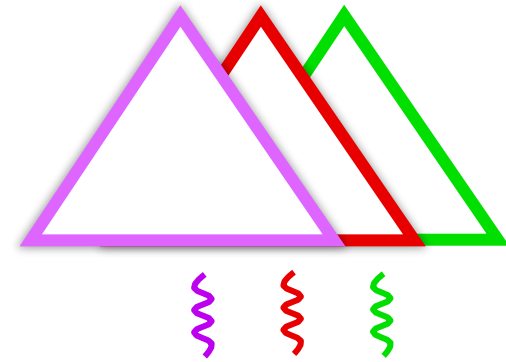
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## Portfolio

[HJS09]

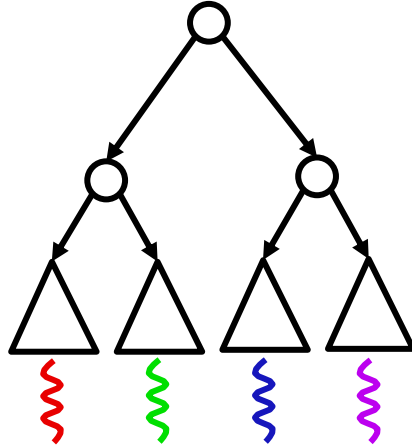


Multiple workers on the whole  
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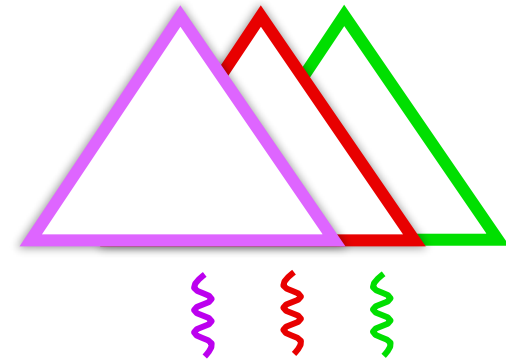
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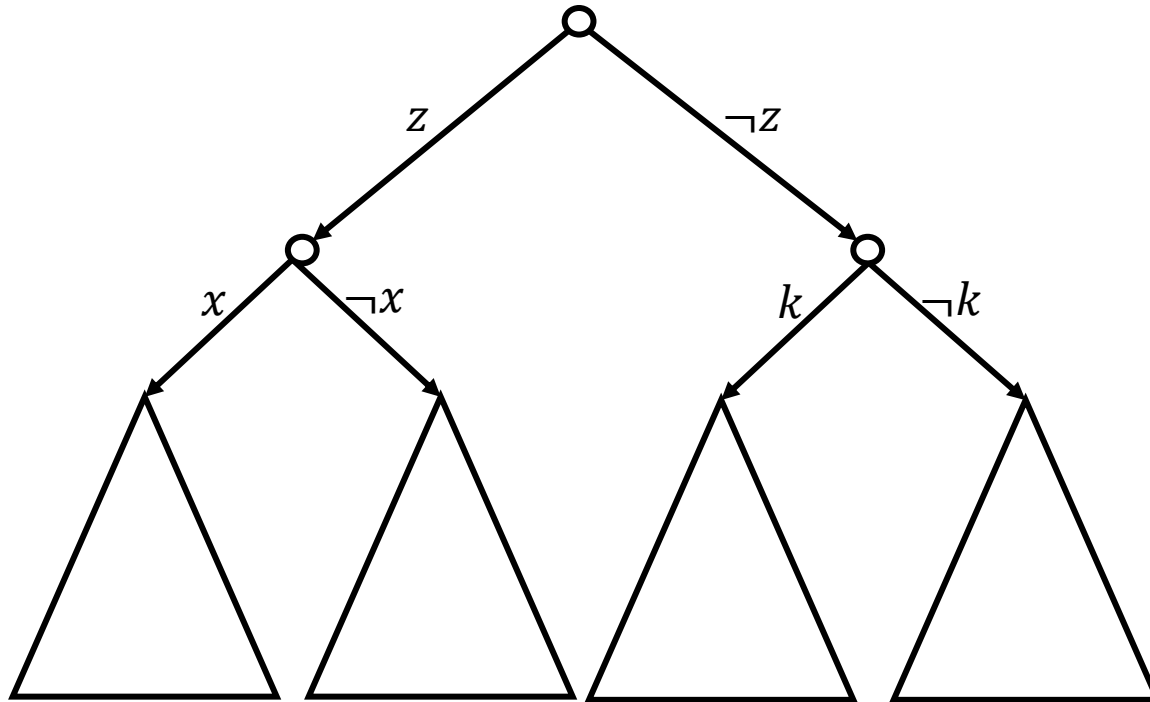


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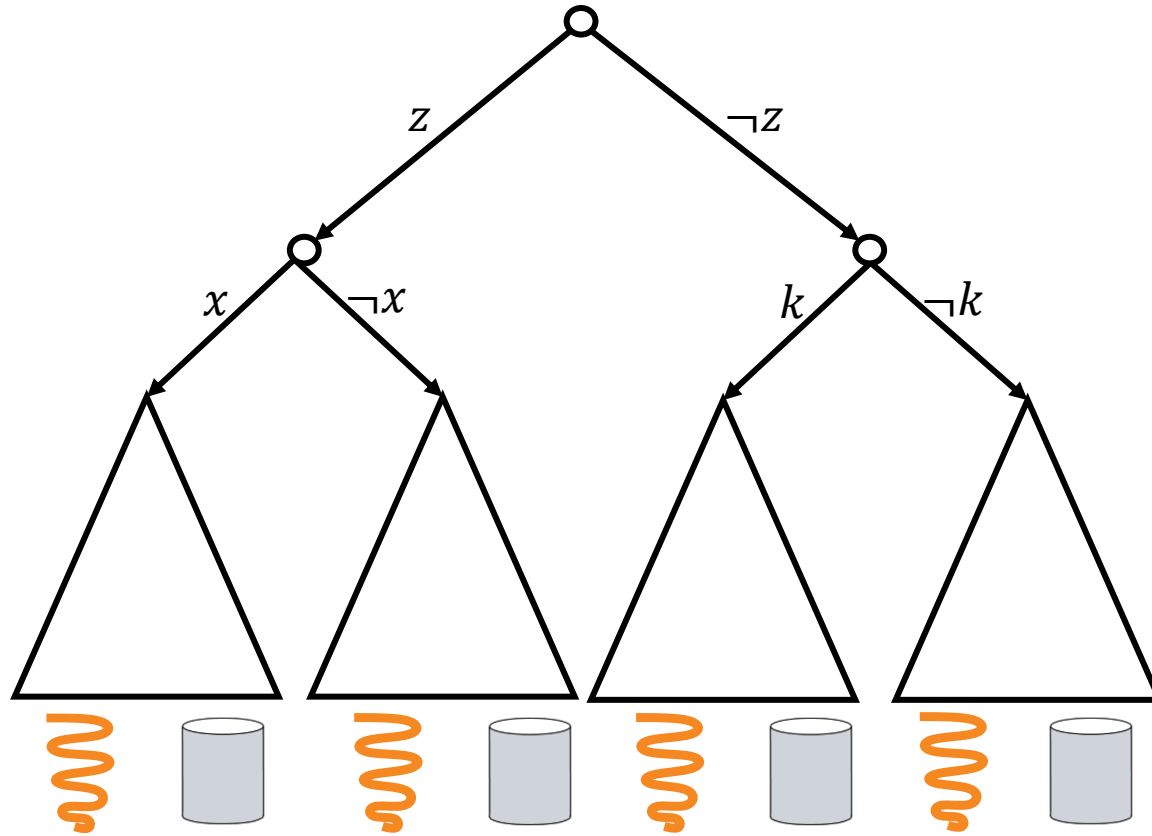
**Information can be shared between the solvers.**



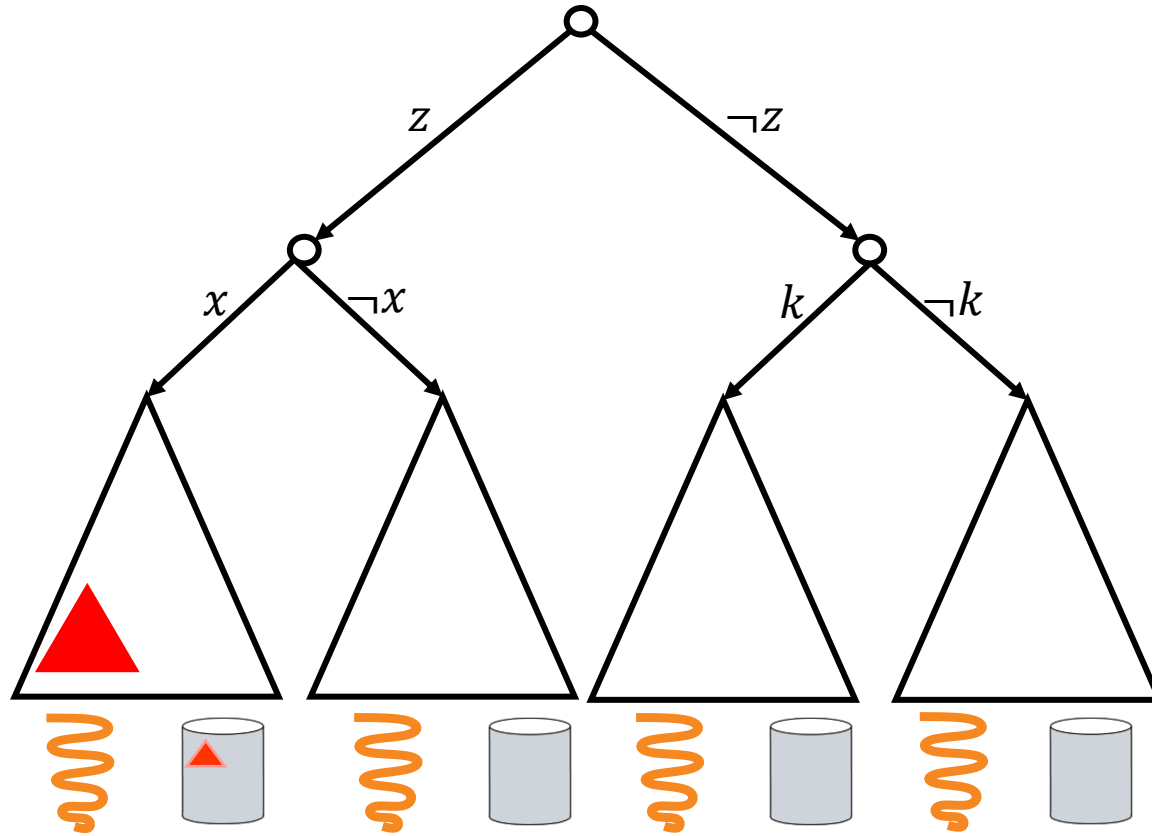
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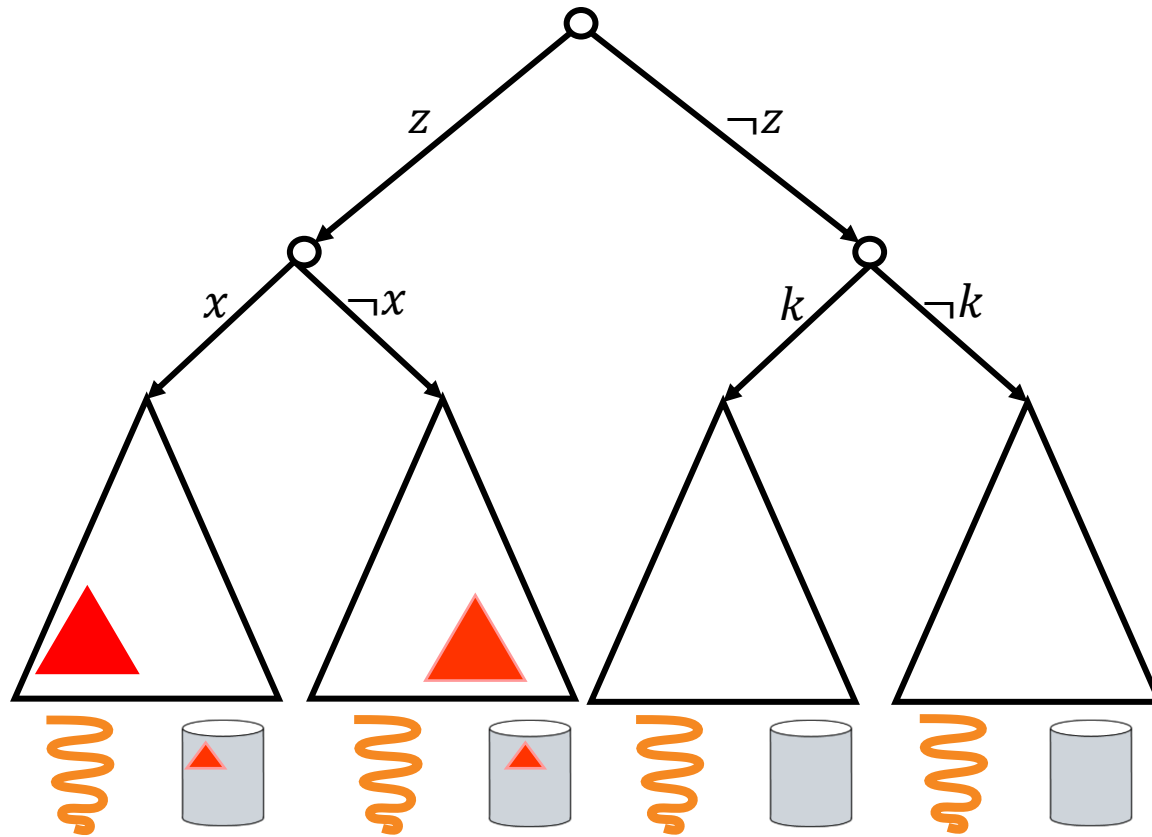
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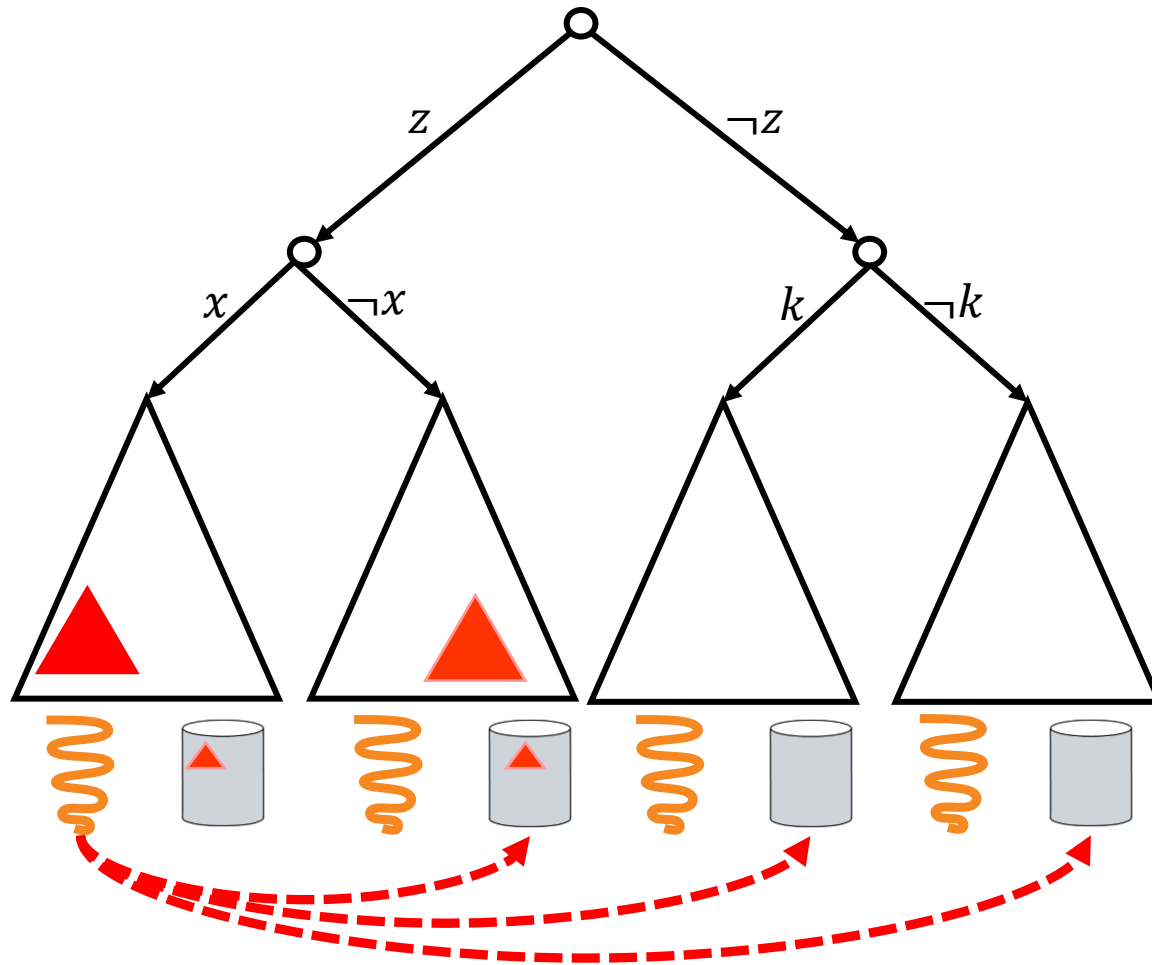
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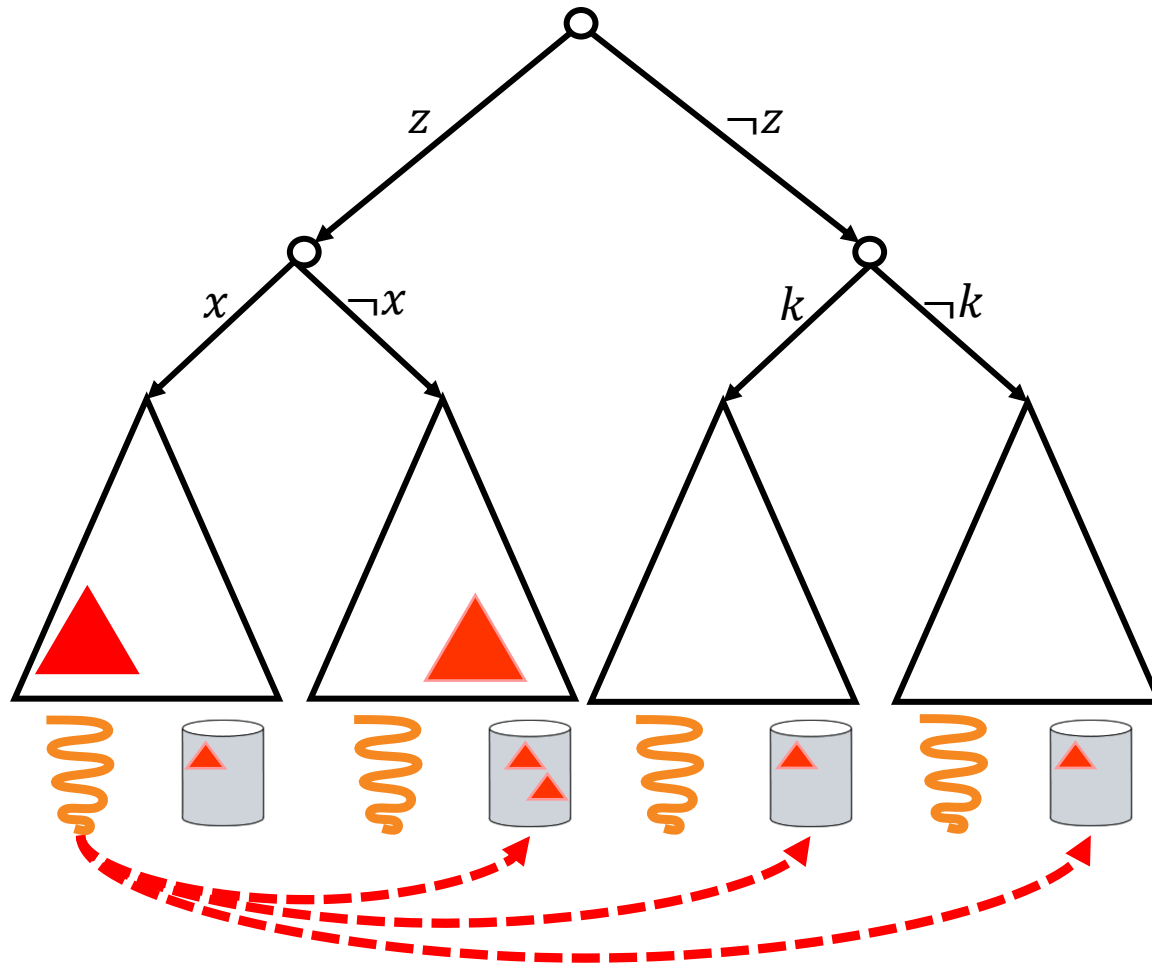
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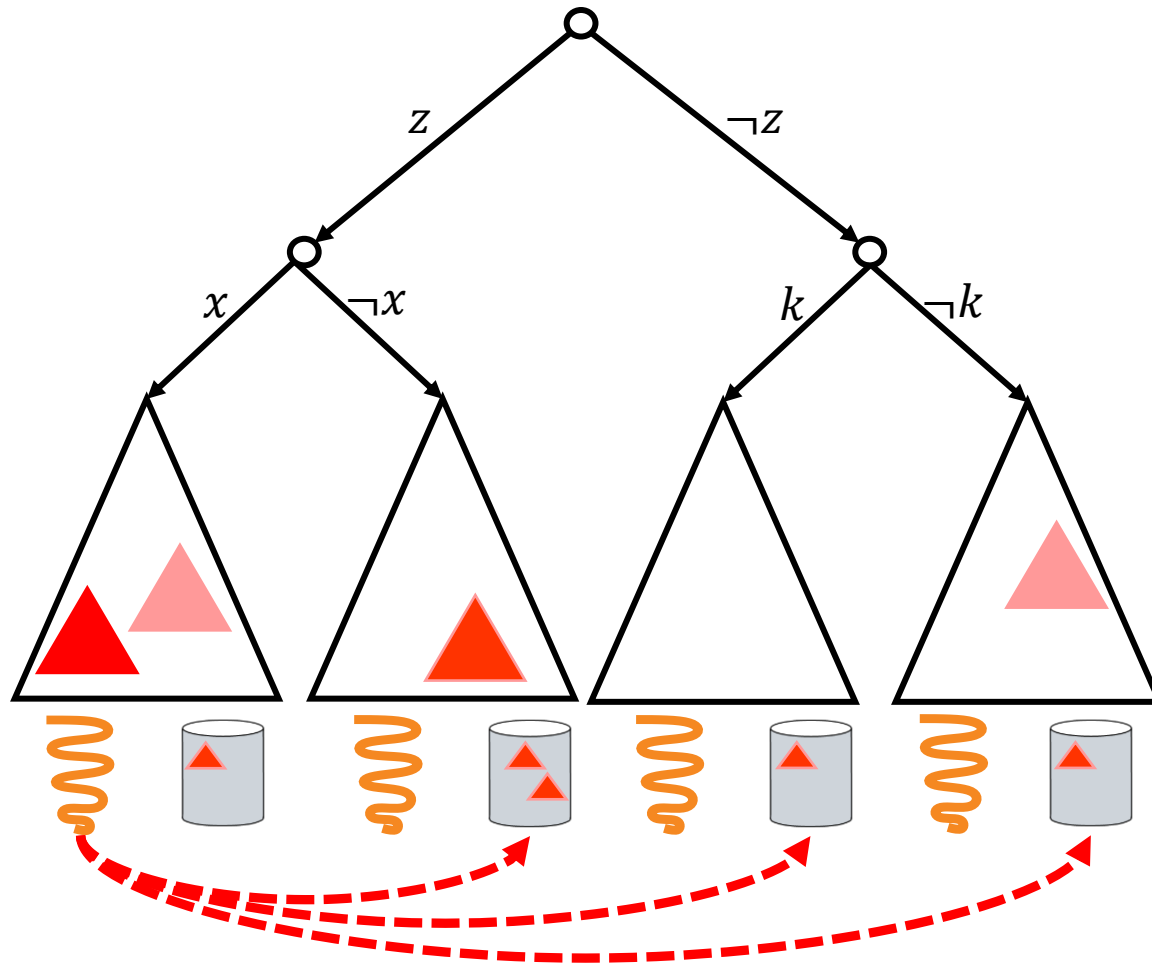
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# Challenges

In practice, sharing too much clauses can impair performances.

## **Algorithmic reasons:**

- slow down unit propagation
- slow down garbage collection

## **Concurrency/Hardware reasons:**

- memory contention (cacheline, alloc)
- memory footprint
- synchronization

**How to select clauses to find the right trade-offs between cost and gain ?**  
**(One of the 7 challenges of parallel SAT solving [[HW13](#)])**



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*What is shared ?*

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**A thread applies the hordesat sharing strategy [[BSS15](#)]:**

- **Try to share 1500 literals**
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Added mechanisms:

- **Bloom filter for exchanged learned clauses [[SS21](#)]**
- **Asynchronous clauses minimization [[VFBSK20](#)]**

# Progressive integration of sharing strategies

P-MCOMSPS sequential engine does not use recent discoveries

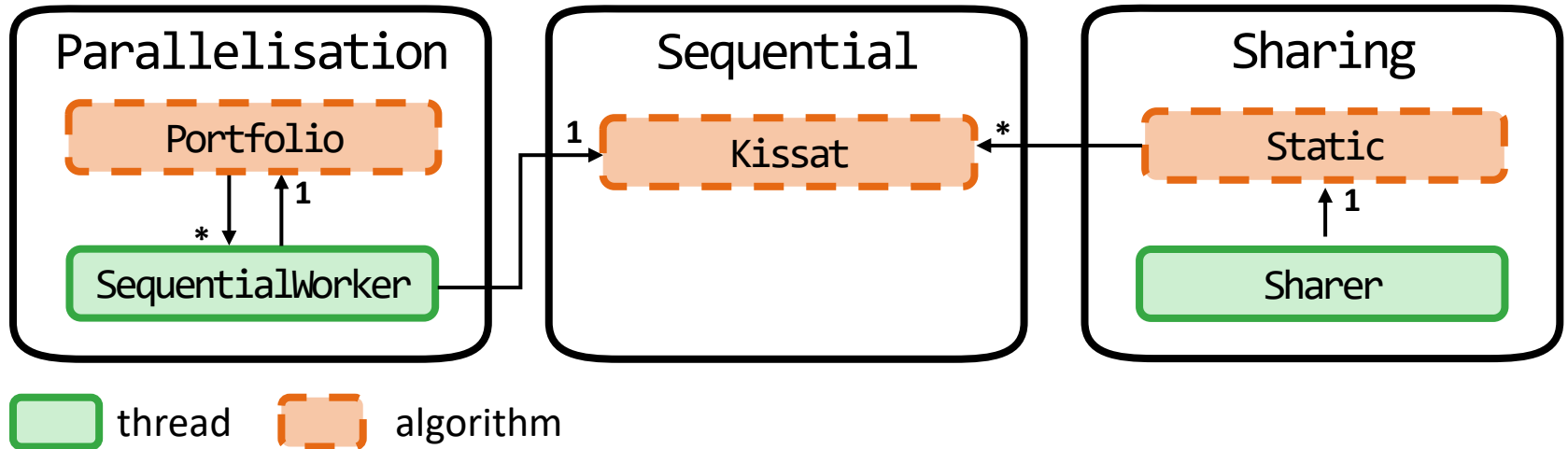
**Kissat [[BFFH20](#)] dominates sequential and parallel solving**

## Contributions:

- **Implementation on another parallel SAT solver:**
  - We incrementally integrated each mechanism
  - We evaluated them on the SAT 2022 competition bench
  - We demonstrated combinations of mechanisms that improved performance
- **Scaling study performed on 48 and 64 core machines:**
  - Shows good scaling for SAT instances
  - Limited results for UNSAT instances

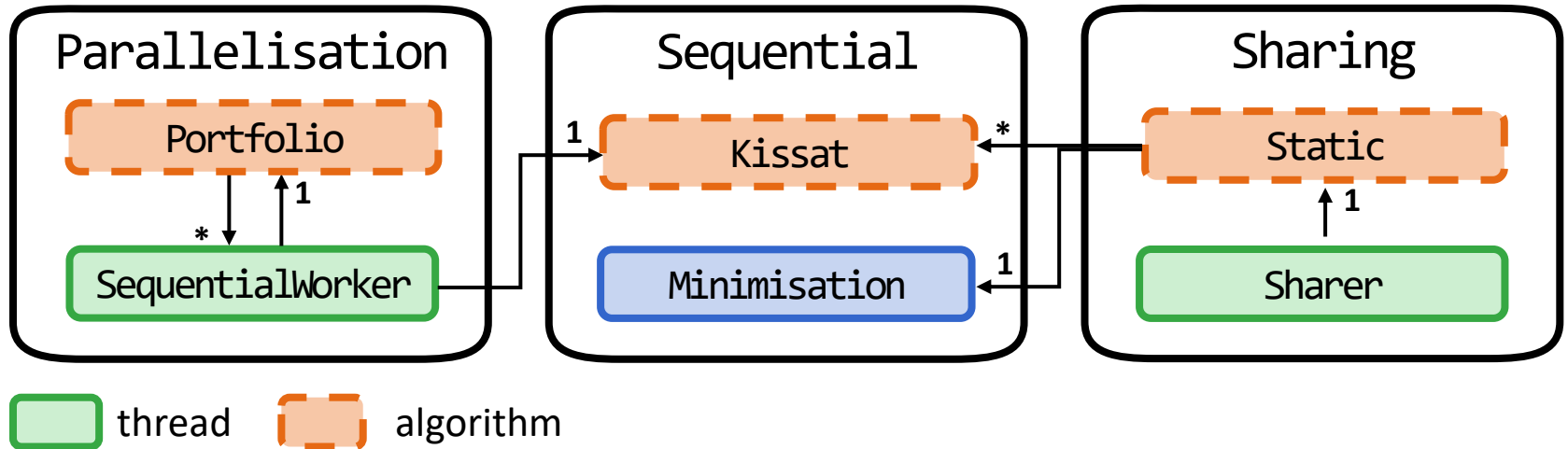
# Incremental integration of sharing components

**Parkissat**: winner of the parallel track 2022 [[ZCC22](#)]



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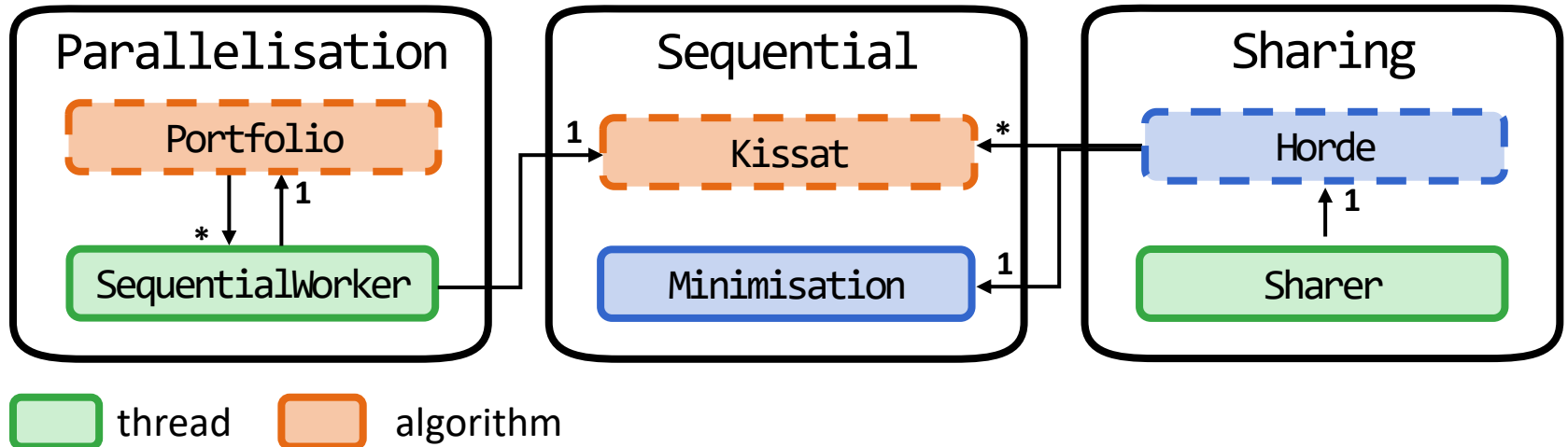
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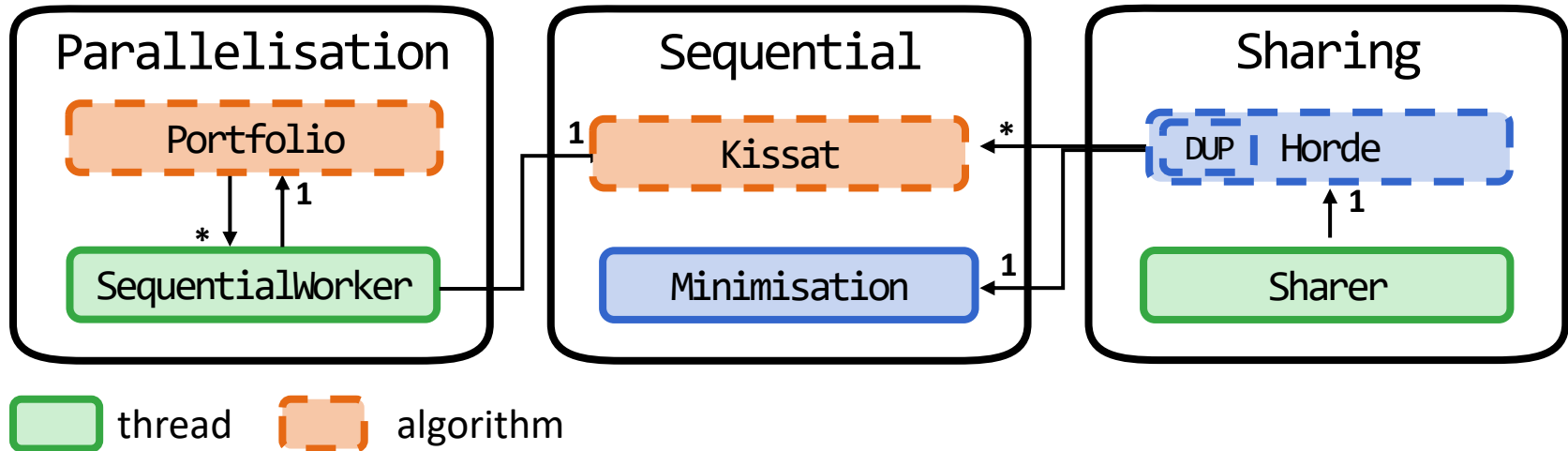


- **STR:** Asynchronous minimisation of clauses
- **Dynamic sharing strategy:** same than P-MCOMSPS



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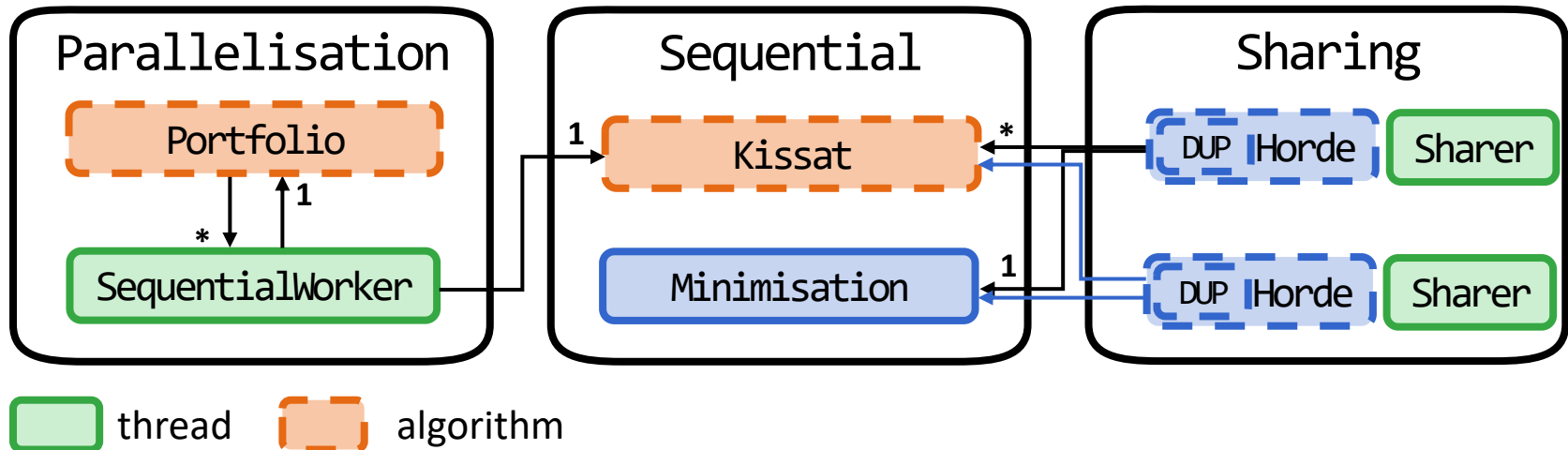
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- **Dup:** Bloom filter for exchanged clauses

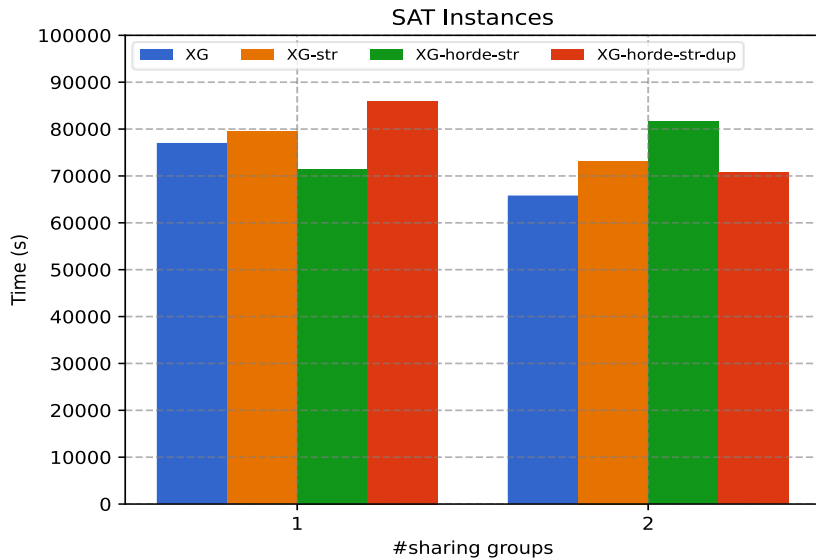
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- **STR:** Asynchronous minimisation of clauses
- **Dynamic sharing strategy:** same than P-MCOMSPS
- **Dup:** Bloom filter for exchanged clauses
- **2G:** Add of a second sharing thread

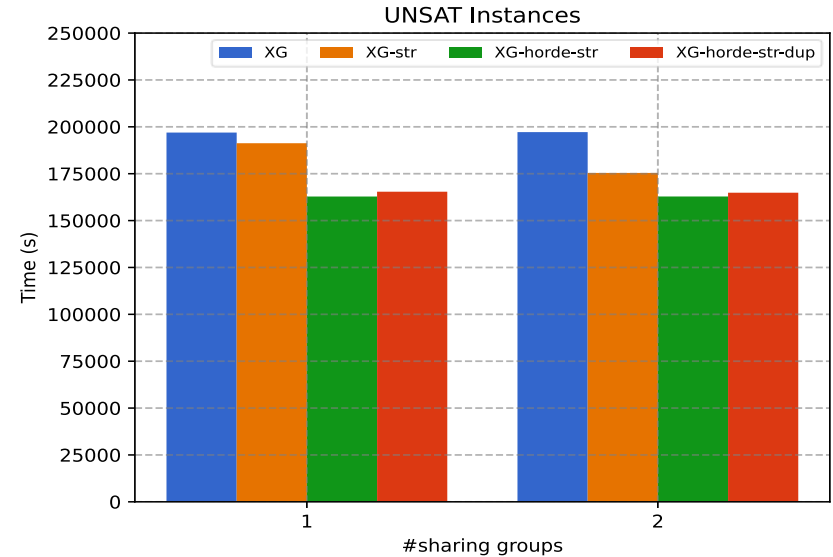
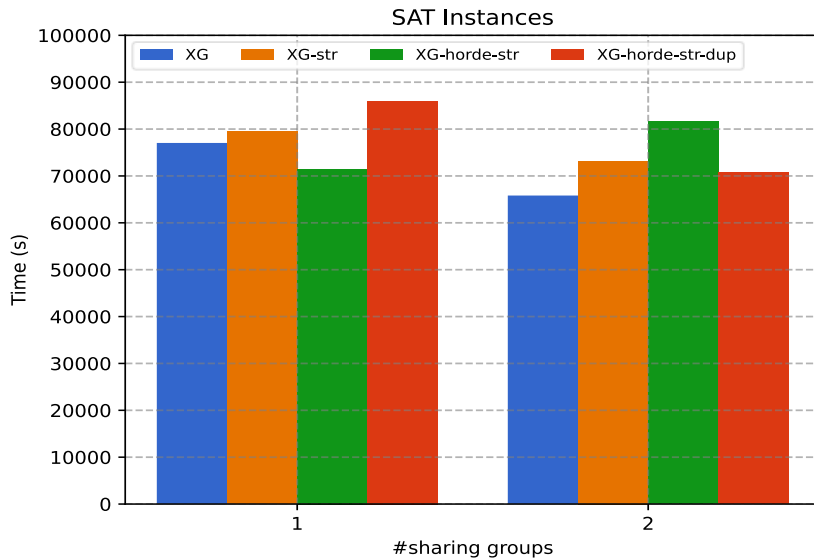
# Performance study



## SAT instances: race to the solution

- Using more threads for sharing is useful
- Other heuristics are not fruitful

# Performance study



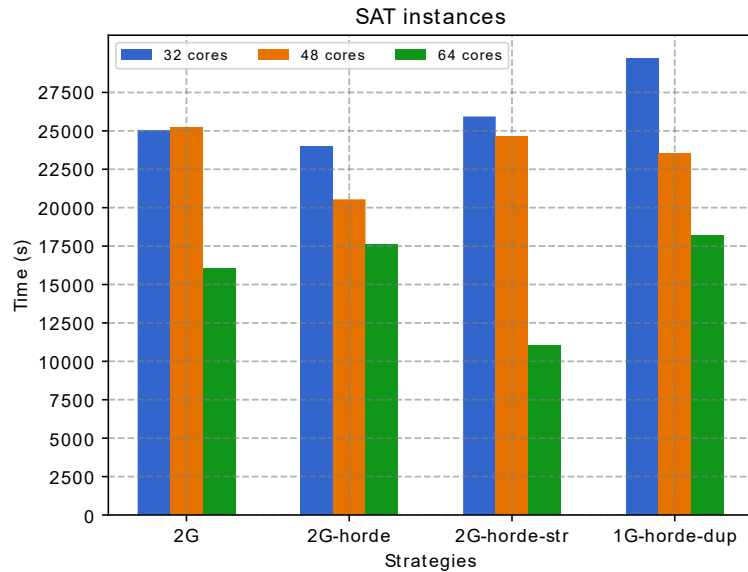
## SAT instances: race to the solution

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## UNSAT instances: race to the unsat core

- Strengthening and horde improve performance
- Duplicates management does not

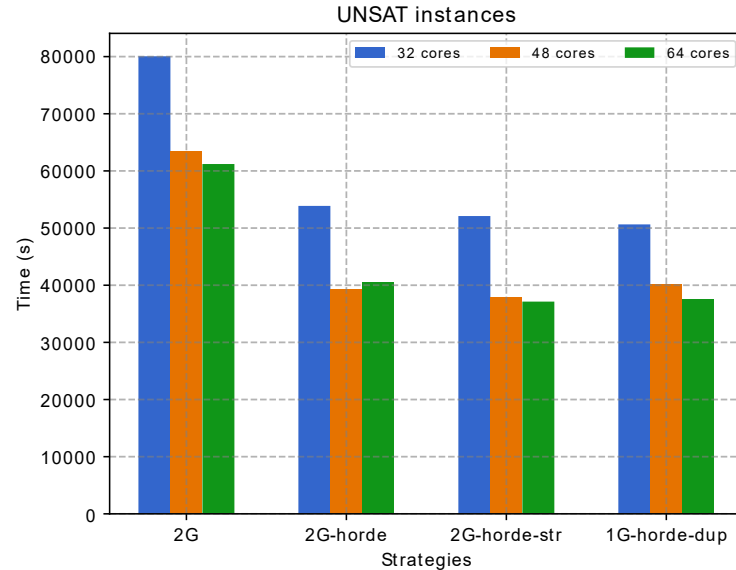
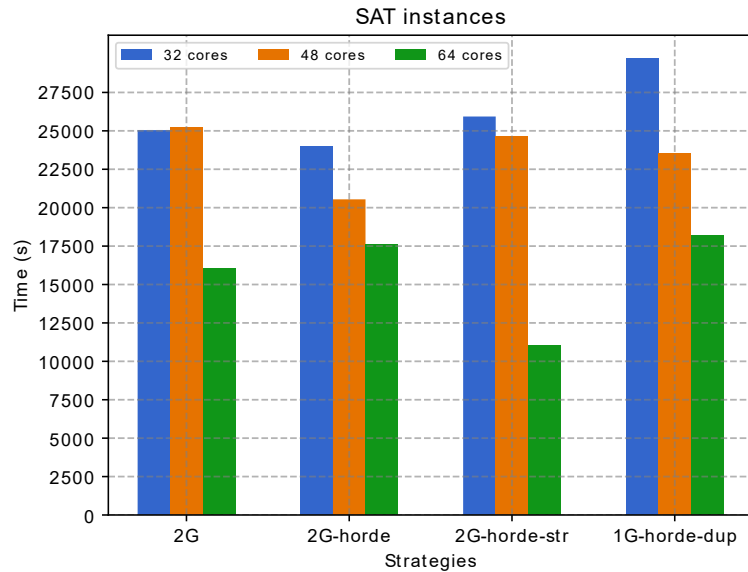
# Scaling study



SAT instances:

**The probability of finding a solution increases with the number of threads.**

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SAT instances:

**The probability of finding a solution increases with the number of threads.**

UNSAT instances:

**Each path must be explored, more computational resources do not remove algorithm limitations.**

# Conclusion

## Contributions :

- Evaluation of multiple sharing strategies
- Boost the performance of the best parallel solver
- Detect a scaling problem in the UNSAT resolution for this solver

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