On Improving the Backjump Level in PB Solvers

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> *Such instances can be solved efficiently with pseudo-Boolean solvers based on cutting planes*

Pseudo-Boolean (PB) Constraints

PB solvers generalize SAT solvers to take into account

- normalized PB constraints $\sum_{i=1}^{n} \alpha_i \ell_i \geq \delta$
- cardinality constraints $\sum_{i=1}^{n} \ell_i \geq \delta$
- clauses $\sum_{i=1}^{n} \ell_i \geq 1 \equiv \bigvee_{i=1}^{n} \ell_i$

in which

- the coefficients α_i are non-negative integers
- ℓ_i are literals, i.e., a variable v or its negation $\bar{v} = 1 - v$
- the degree δ is a non-negative integer

Cutting Planes for CDCL

The generalized resolution proof system [Hooker, 1988] is used in PB solvers as the counterpart of the resolution proof system:

$$\frac{\alpha\ell + \sum_{i=1}^{n} \alpha_i \ell_i \geq \delta_1 \qquad \beta\overline{\ell} + \sum_{i=1}^{n} \beta_i \ell_i \geq \delta_2}{\sum_{i=1}^{n}(\beta\alpha_i + \alpha\beta_i)\ell_i \geq \beta\delta_1 + \alpha\delta_2 - \alpha\beta} \text{ (cancellation)}$$

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Using these rules during conflict analysis requires to apply additional operations to preserve CDCL invariants

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> *However, learning this constraint is not optimal in general in terms of backjump level*

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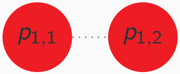

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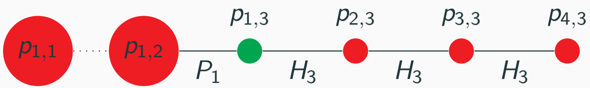

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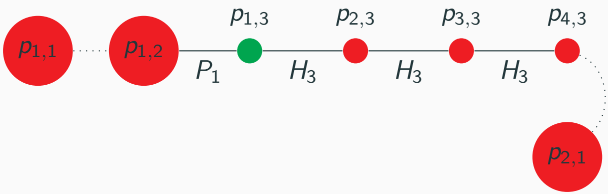

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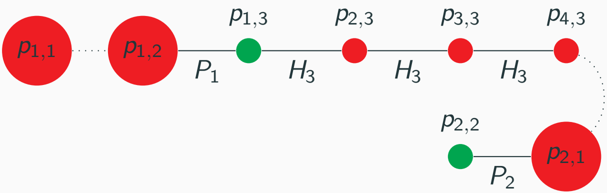

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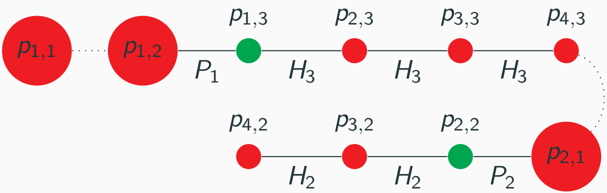

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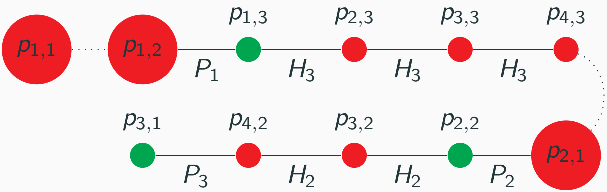

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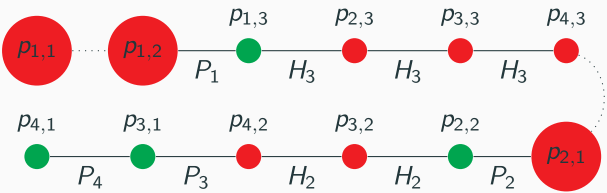

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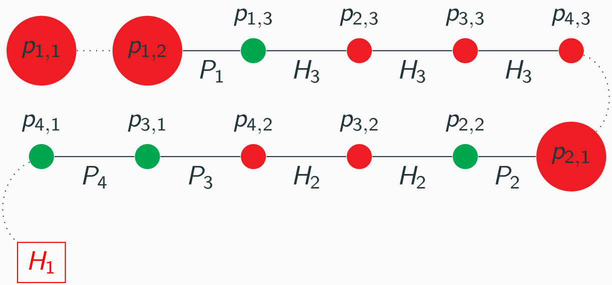

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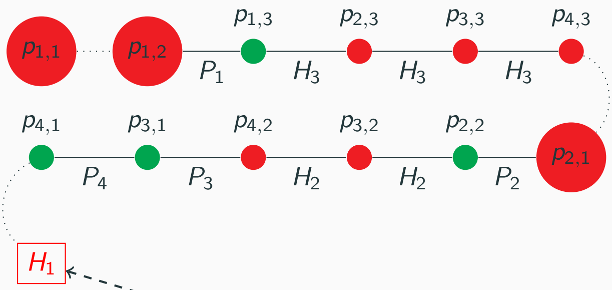

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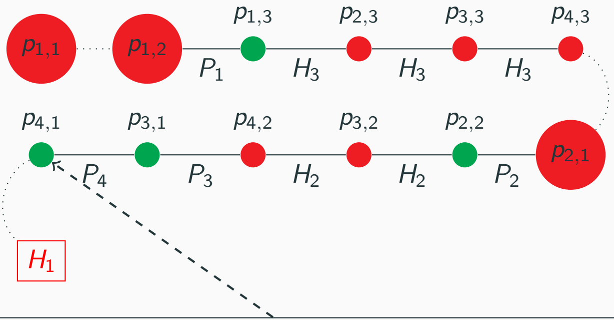

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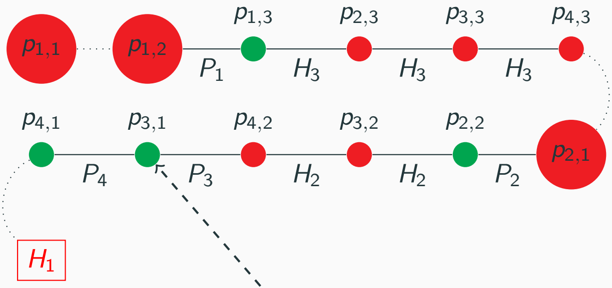

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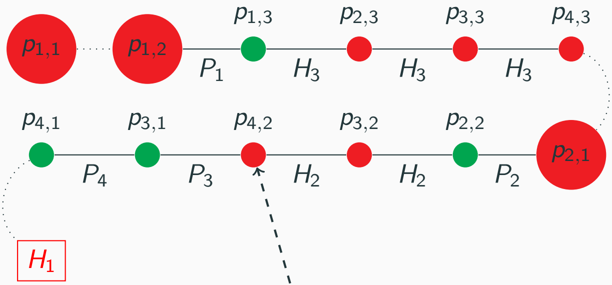

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Motivation: A Pigeon-Hole Principle Example

$$H_1 \equiv \bar{p}_{1,1} + \bar{p}_{2,1} + \bar{p}_{3,1} + \bar{p}_{4,1} \geq 3$$
$$H_2 \equiv \bar{p}_{1,2} + \bar{p}_{2,2} + \bar{p}_{3,2} + \bar{p}_{4,2} \geq 3$$
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$$P_1 \equiv p_{1,1} + p_{1,2} + p_{1,3} \geq 1$$
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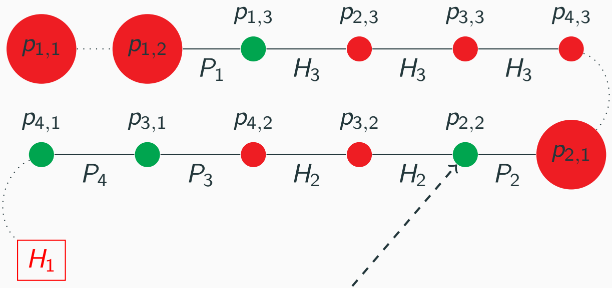

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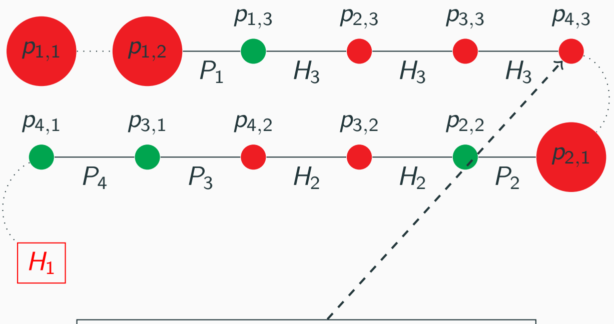

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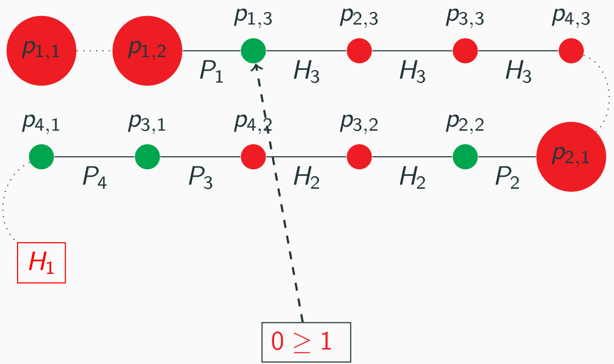

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The rule is applied iteratively until a propagation at the best assertion level found so far is restored

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We also tried different additional criteria for improving the efficiency of the approach but in practice they are used very rarely

Experiments in *Sat4j*: Sub-Optimal Analyses

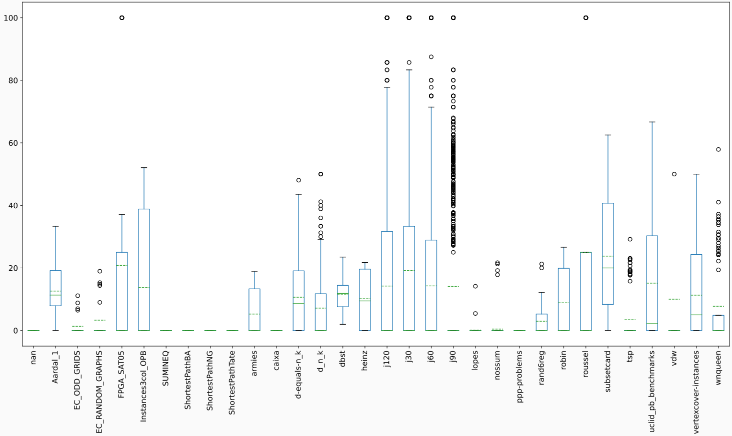

Figure 1: Boxplots of the percentage of sub-optimal analyses per family.

Experiments in *Sat4j*: Conflicts

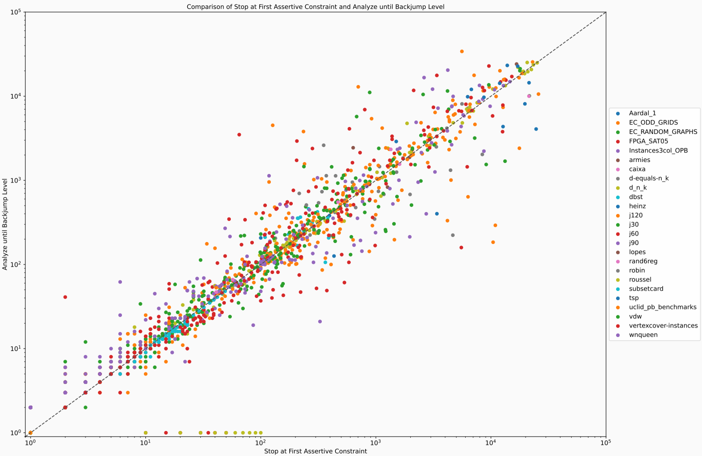

Figure 2: Scatter plot of the number of conflicts

Experiments in *Sat4j*: Cancellations

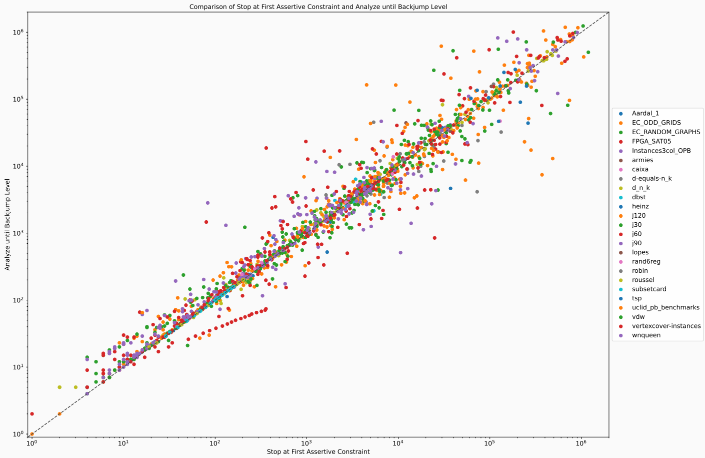

Figure 3: Scatter plot of the number of cancellations

Experiments in *Sat4j*: Runtime

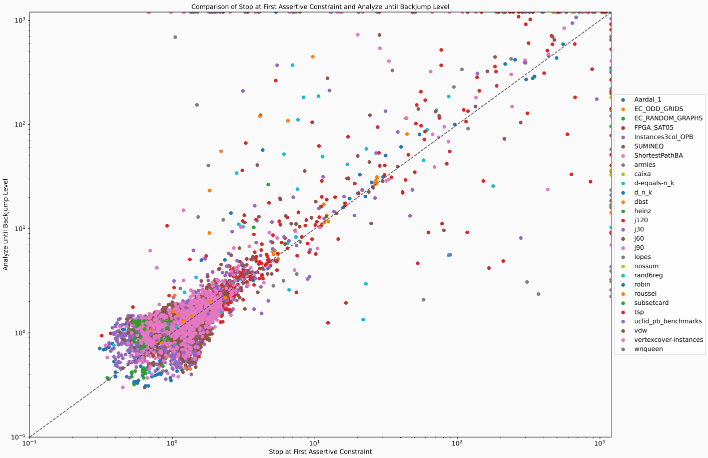

Figure 4: Scatter plot of the runtime

Conclusion and Perspectives

- Current PB solvers inherit the CDCL architecture of modern SAT solvers by implementing cutting planes rules
- However, some invariants of CDCL are broken in PB solvers, such as the optimality of the 1-UIP
- We presented different strategies for continuing the analysis, while guaranteeing to improve the backjump level

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- However, some invariants of CDCL are broken in PB solvers, such as the optimality of the 1-UIP
- We presented different strategies for continuing the analysis, while guaranteeing to improve the backjump level

- Improve the efficiency of the proposed approaches
- Find better ways to decide when to stop (e.g., based on the quality of the learned constraint)
- Use speculative techniques to guess when the analysis should stop, while allowing to continue the analysis asynchronously
- Consider the use of chronological backtracking techniques

On Improving the Backjump Level in PB Solvers

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