A Study on Implied Constraints in a MaxSAT Approach to B2B Problems

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The B2B Scheduling Optimization Problem (I)

- A set of **Participants** $\mathcal{P}$
- A set of **Meetings** $\mathcal{M} \subset \mathcal{P} \times \mathcal{P}$
- **Accommodation capacity**:
  - A set of **Time Slots** $\mathcal{T}$
  - A set of **Locations** $\mathcal{L}$
- Meetings restrictions:
  - Forbidden time slots
  - Morning/Afternoon meetings

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B2B Scheduling Problem (B2BSP)

Finding a total **mapping** from M to $T \times L$, without overlapping of meetings in time nor in location and respecting the meetings restrictions.

The B2B Scheduling Optimization Problem (II)

Feasibility of a Schedule

- Each participant has at most one meeting scheduled in each time slot.
- Every meeting is scheduled respecting meeting restrictions for any of its participants.
- At most one meeting is scheduled in a given time slot and location.
- Each meeting is scheduled in one and only one time slot.

The B2B Scheduling Optimization Problem (III)

Idle Time Periods

9:00	9:20	9:40	10:00	10:20	10:40	11:00	12:20	12:40	13:00
free	m_1	free	free	free	m_2	free	free	m_3	free
		hole 1				hole 2			

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B2BSOP

. . . **minimizing** the total number of **idle time periods** **cumulatively** over all participants.

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B2BSOP

. . . **minimizing** the total number of **idle time periods** **cumulatively** over all participants.

B2BSOP-h

. . . such that the **difference** between the number of idle time periods of the participants is **at most** a given parameter h (**homogeneity**).

B2B Encodings

Bofill, Espasa, Garcia, Palahí, Suy, Villaret. CP 2014.

- CP encoding.
- PB encoding.

Pesant, Rix, and Rousseau. CPAIOR 2015.

- MIP encoding.
- Global constraints (MIP and CP)

Bofill, Garcia, Suy, and Villaret. CPAIOR 2015.

- MaxSAT encoding.

MaxSAT Encoding

[Bofill, Garcia, Suy, Villaret. CPAIOR15]

- $\textit{schedule}_{i,j}$: meeting i is scheduled in time slot j
- $\textit{usedSlot}_{p,j}$: participant p has a meeting at time slot j
- $\textit{fromSlot}_{p,j}$: participant p has a meeting at time slot j or before
- *tableCount* : at most one meeting scheduled in a time slot and location
- *max*/*min*/*diff* : homogeneity

Implied Constraints (I)

[Bofill, Garcia, Suy, Villaret. CPAIOR15]

Implied Constraint 1: The number of meetings of a participant p (derived from $usedSlot_{p,j}$) must match the total number of meetings of p.

$$exactly(|meetings(p)|, \{usedSlot_{p,j} \mid j \in \mathcal{T}\}) \quad \forall p \in \mathcal{P}$$

Implied Constraints (II)

[Bofill, Garcia, Suy, Villaret. CPAIOR15]

Implied Constraint 2: The number of participants having a meeting in a given time slot is bounded by twice the number of available locations.

$$atMost(2 \times nTables, \{usedSlot_{p,j} \mid p \in \mathcal{P}\}) \quad \forall j \in \mathcal{T}$$

Benefits of Implied Constraints

[Bofill, Garcia, Suy, Villaret. CPAIOR15]

"*. . . extending the model with implied constraints . . . , we can significantly improve the solving time.*"

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Number of B2B instances used:

- 5 real-world instances.
- 15 instances crafted from them.

Contributions

- **When** is **beneficial** the use of implied constraints?
- Does it **depend** on any **feature** of the problem?

Discussion:

- Can we use this analysis to better understand the efficiency of MaxSAT solvers?
- Can we extract general conclusions on the use of implied constraints to be applied in other problems?

Regular model

The **probability** that any participant **request** a meeting with another is exactly $U \in [0, 1]$.

B2B Random Generator

Regular model

The **probability** that any participant **request** a meeting with another is exactly $U \in [0, 1]$.

- The number of meetings requested by each participant follows a binomial distribution $B(n, p)$, with $n = P - 1$ and $p = U$
- If $(P - 1)U \gg T$, instance infeasible
- Similar number of requests by each participant?
- No meeting restrictions

Definitions

- **Density** d: ratio meetings / accommodation capacity.

$$d = \frac{M}{T \cdot L}$$

- **Shape** s: ratio accommodation capacity.

$$s = \frac{T}{L}$$

Experimental Evaluation

Instances Generation:

- 16 different configurations of density/shape
- 20 random instances per configuration
- **320 different random instances**

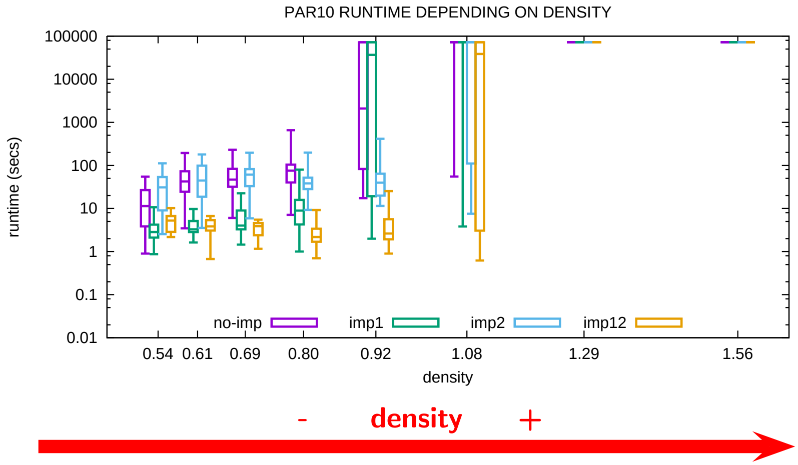
Encodings:

- No implied constraint (*no-imp*)
- Implied Constraint 1 (*imp1*)
- Implied Constraint 2 (*imp2*)
- Both Implied Constraints (*imp12*)

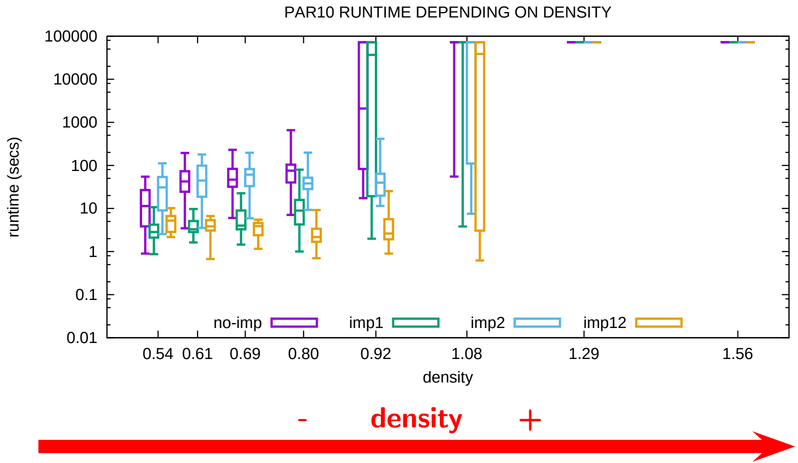
Solving:

- Solved with **Open-WBO**
- Timeout 2h

Varying the Density



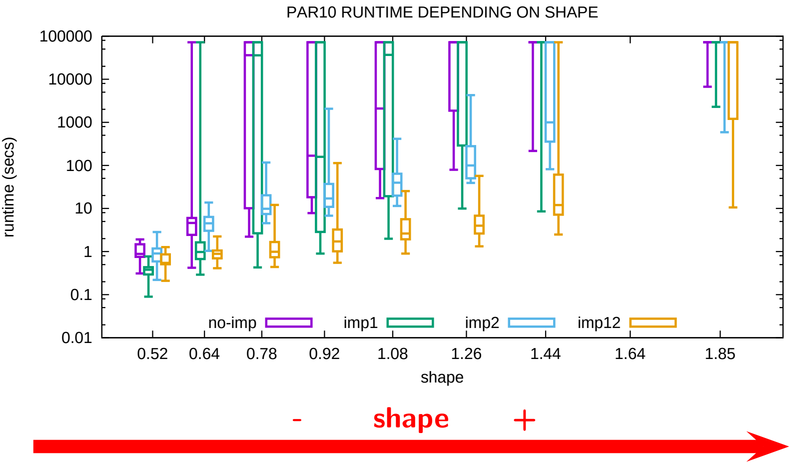
Varying the Density



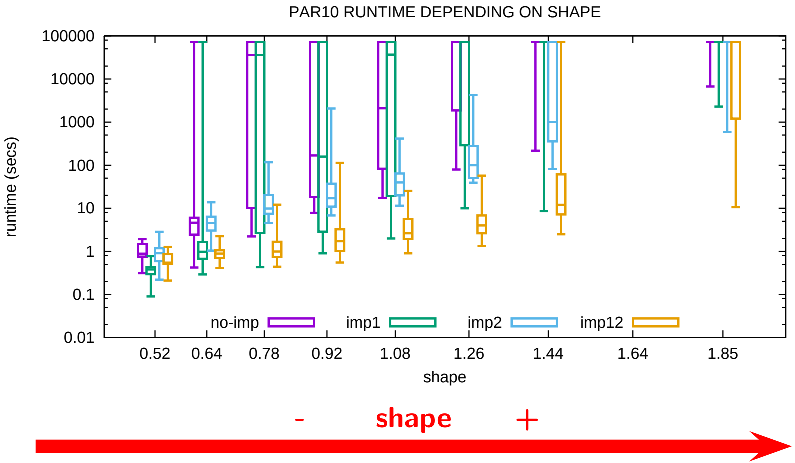
Observation 1

imp1 ⇝ small density. *imp2* ⇝ high density.
Overall, *imp12* always beneficial.

Varying the Shape



Varying the Shape



Observation 2

imp1 ⇝ small shape. *imp2* ⇝ high shape.
Overall, *imp12* always beneficial.

Checking the Observations in Real-World Instances

- **Modify** real-world B2B instances.
  - Smaller number of perturbations (in T and L).

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- **Modify** real-world B2B instances.
  - Smaller number of perturbations (in T and L).

- **Observation 1**: *imp1* $\rightsquigarrow$ small density, *imp2* $\rightsquigarrow$ high density, *imp12* always beneficial.
  - Seems to be valid in real-world instances.

- **Observation 2**: *imp1* $\rightsquigarrow$ small shape, *imp2* $\rightsquigarrow$ high shape, *imp12* always beneficial.
  - Possibly valid in real-world instances. More doubts...

Performance of the MaxSAT Solver (I)

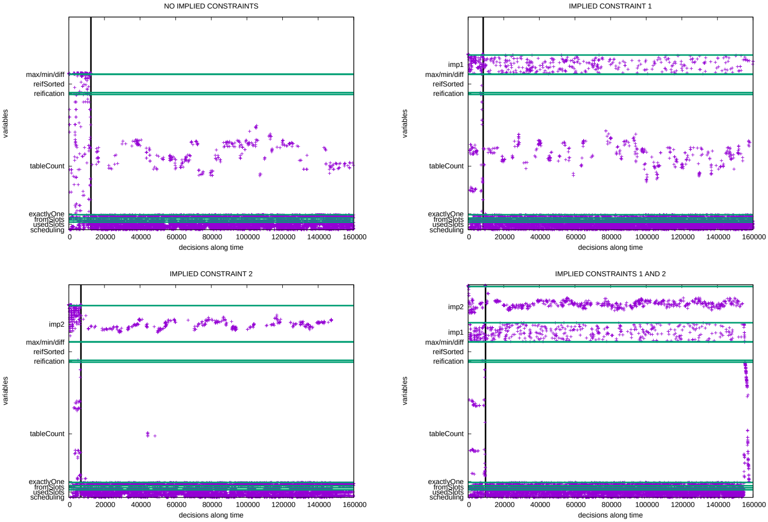
- **Open-WBO**: MSU3 algorithm (UNSAT-based)
- Internally using **Glucose** (**CDCL**)
  - Search: guided by the conflicts.

Performance of the MaxSAT Solver (II)

- Random instance with **low density** (general results for all instances analyzed).
- According to Obs. 1, *imp1* is faster than *imp2*, and *imp12* is faster overall.

	Runtime	Decisions
No implied constraints:	70.78	3444288
Implied constraint 1:	5.88	343487
Implied constraint 2:	84.16	3152049
Implied constraints 1 and 2:	**3.07**	**157729**

Performance of the MaxSAT Solver (III)



Performance of the MaxSAT Solver (IV)

instance	decisions on		
	imp1	*imp2*	*imp12*
random low density	3.90%		6.32%

Performance of the MaxSAT Solver (IV)

- The use of **both implied constraints** **reinforces** the decisions on the most *efficient* implied constraint (dependent on the problem), reducing the solving runtime.

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	decisions on		
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random high density		12.44%	17.86%

Performance of the MaxSAT Solver (IV)

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instance	decisions on *imp1*	decisions on *imp2*	*imp12*
random low density	3.90%		6.32%
random high density		12.44%	17.86%
forum-14	2.54%		3.55%
tic-14crafd		4.93%	5.18%

Conclusions and Future Work

Conclusions:

- Random B2B instances generator.
- Strengths and weaknesses of using **implied constraints**.
- Effectiveness dependent on characteristics of the instance: **density** and **shape**.
- Duality in the benefits of using **both**.

Future Work:

- Heuristics to prioritize decisions: *imp1* or *imp2*.
- A more realistic generator: requests, forbidden time slots, . . .
- Other implied constraints.

Discussion

- Can we use this analysis to better understand the efficiency of MaxSAT solvers?
- Can we extract general conclusions on the use of implied constraints to be applied in other problems?

- **Redundant information** vs **Compact encoding**
  - Faster propagations
  - Faster detections of conflicts
  - Better pruning

Thank for your attention

Questions?