Seeking Practical CDCL Insights from Theoretical SAT Benchmarks

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Joint work with Jan Elffers, Karem Sakallah, and Laurent Simon

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- And a sometimes somewhat bewildering alphabet soup of heuristics (VSIDS, 1UIP, LBD, BCD, BCE, BVA, ELS, FLP, VE, VMTF, . . .)
- Want a **deeper understanding** of how these solvers actually work

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- Run CDCL with different heuristics to see how performance affected
- Benchmarks extremal w.r.t. different properties — can be expected to "challenge" solver

This Talk

- Describe candidate set of benchmarks
- Discuss CDCL parameter configurations to be tested
  (focus on basic CDCL search, not preprocessing techniques)
- Report on some preliminary findings
  Warning for sensitive viewers: will be plots, but no cactus plots
- **Caveat:** Still very much work in progress
  Hope that presentation can generate interesting discussions

Some Notation and Terminology

- Literal a: variable x or its negation $\overline{x}$ (or $\neg x$)
- Clause $C = a_1 \vee \cdots \vee a_k$: disjunction of literals
  (Consider as sets, so no repetitions and order irrelevant)
- CNF formula $F = C_1 \wedge \cdots \wedge C_m$: conjunction of clauses
- k-CNF formula: CNF formula with clauses of size $\leq k$
  (where k is some constant)
- N denotes size of formula (# literals counted with repetitions)
- $\mathcal{O}(f(N))$ grows at most as quickly as $f(N)$ asymptotically
  $\Omega(g(N))$ grows at least as quickly as $g(N)$ asymptotically
  $\Theta(h(N))$ grows equally quickly as $h(N)$ asymptotically

Proof System Underlying CDCL: Resolution

Goal: refute **unsatisfiable** CNF

Start with clauses of formula (axioms)

Derive new clauses by resolution rule

$$\frac{C \vee x \qquad D \vee \overline{x}}{C \vee D}$$

Proof ends when empty clause $\perp$ derived

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- annotated list or
- directed acyclic graph

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3.	$\overline{x} \lor z$	Axiom
4.	$\overline{y} \lor \overline{z}$	Axiom
5.	$\overline{x} \lor \overline{z}$	Axiom
6.	$x \lor \overline{y}$	Res$(2, 4)$
7.	x	Res$(1, 6)$
8.	$\overline{x}$	Res$(3, 5)$
9.	$\perp$	Res$(7, 8)$

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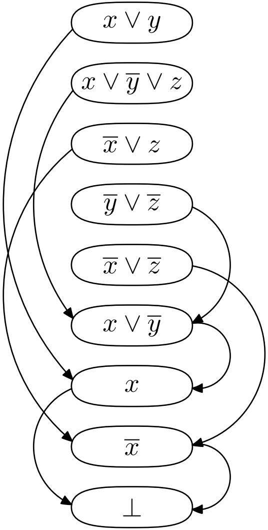

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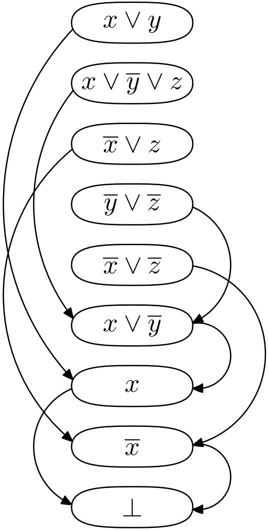

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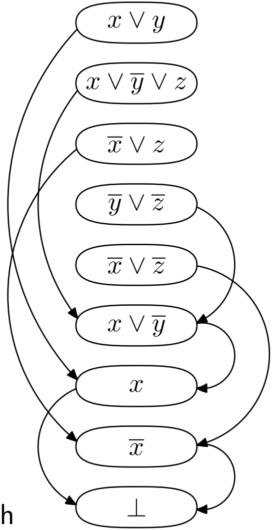
Tree-like if DAG is tree (corresponds to DPLL)
Regular if resolved variables don't repeat on path



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Size/length of proof $=$ $\#$ clauses (9 in example on previous slide)
Length of refuting F $=$ min over all proofs for F

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Most fundamental measure in proof complexity

Lower bound on CDCL running time*
(can extract resolution proof from execution trace)

Never worse than $\exp(\mathcal{O}(N))$

Matching $\exp(\Omega(N))$ lower bounds known [Urq87, CS88, BW01]

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(*) Ignores preprocessing — focus here on CDCL proof search

Resolution Space

Space = max # clauses in memory when performing refutation

Motivated by SAT solver memory usage (but also intrinsically interesting for proof complexity)

Can be measured in different ways — makes most sense here to focus on clause space

Space at step t = # clauses at steps $\leq t$ used at steps $\geq t$

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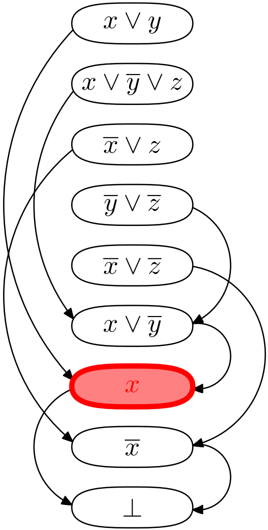

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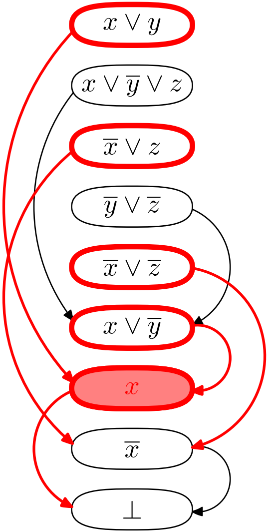

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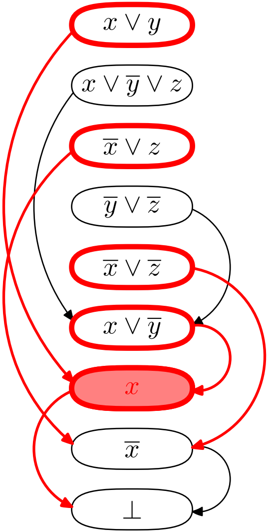

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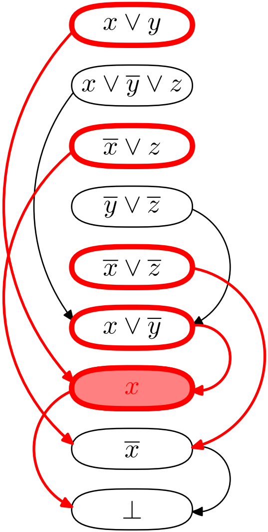

Bounds on Resolution Space

Space always at most $N + \mathcal{O}(1)$ (!) [ET01]

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But:

- Hold even for optimal algorithms that magically know exactly which clauses to throw away or keep
- So significantly more space might be needed in practice
- And linear space upper bound obtained for proofs of exponential size

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Really strong width lower bounds $\Rightarrow$ length lower bounds [BW01]

But only moderately strong width lower bounds don't imply anything for length [BG01] (except hardness for tree-like resolution / DPLL)

Collection of Combinatorial Benchmarks

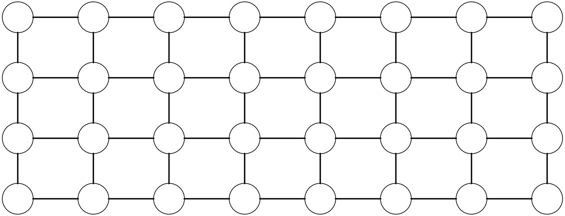
1. Tseitin formulas [Tse68, Urq87]
2. Ordering principle formulas [Kri85, Stå96]
3. Pebbling formulas [BW01, BN08]
4. Stone formulas [AJPU07]
5. Zero-one designs / subset cardinality formulas [Spe10, VS10, MN14]
6. Even colouring formulas [Mar06]
7. Relativized pigeonhole principle (RPHP) formulas [AMO13, ALN16]

Some General Comments on Benchmarks

- Tweak instances so that all have short resolution proofs (even linear size for all except relativized RPHP)
  - proofs can in principle be found by CDCL
  - without any preprocessing
  - often even without any restarts
  - sometimes even without learning, i.e., just DPLL (though might incur some blow-up)
  - ... given right variable decision order
- Test theoretical results in [AFT11, PD11]: Does CDCL search for proofs efficiently?
- Several benchmarks extremal w.r.t. proof complexity measures or trade-offs between measures (see workshop paper for details)
- Practical note: many (though not all) instances generated using **CNFgen** [CNF, LENV16]

Tseitin Formulas

- Take $w \times m$ grid, $w \ll m$



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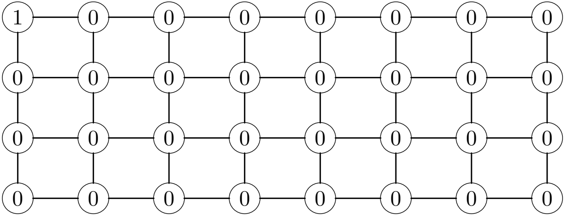

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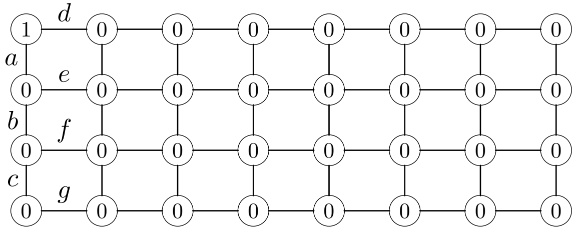

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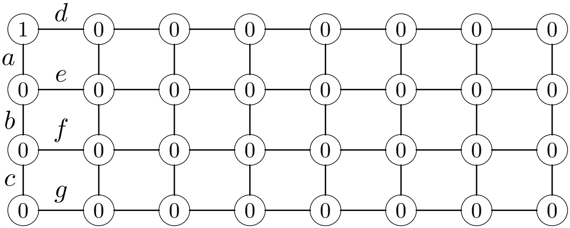
  "vertex label = parity of incident edges"



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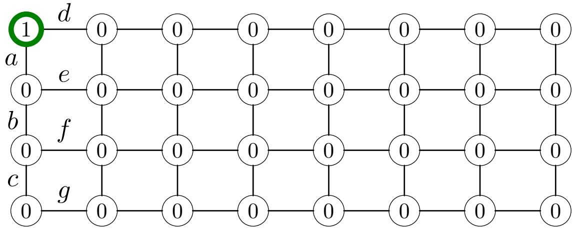

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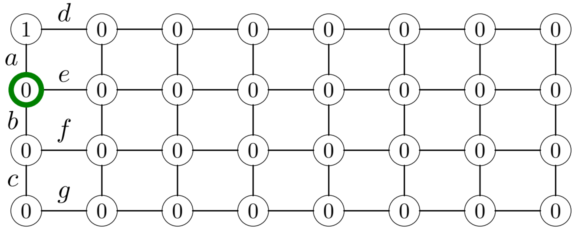

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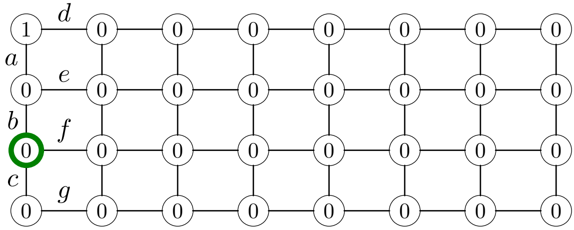

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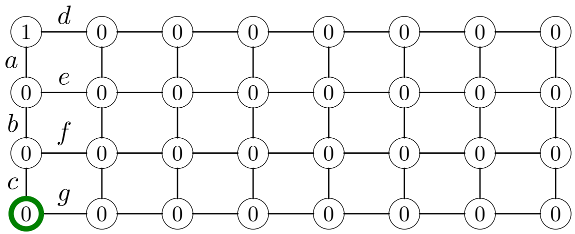

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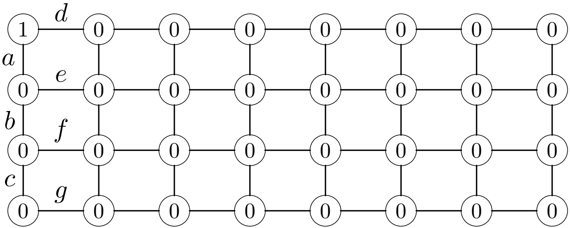
- Hard for well-connected graphs [Urq87] but easy on grids with $w = \mathcal{O}(1)$ (even for DPLL)



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\land\ & (\overline{c} \lor g) \\
& \vdots
\end{aligned}
$$

Subset Cardinality Formulas / Zero-One Designs

Proposed by [Spe10, VS10]

Variables $=$ 1s in matrix with four 1s per row/column $+$ extra 1

Each row wants majority true; each column wants majority false

$$\begin{pmatrix}
\mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\
0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\
0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 \\
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0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\
\mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 & \mathbf{1}
\end{pmatrix}$$

$$\begin{aligned}
& (x_{1,1} \lor x_{1,2} \lor x_{1,4}) \\
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\end{aligned}$$

Subset Cardinality Formulas / Zero-One Designs

Proposed by [Spe10, VS10]

Variables = 1s in matrix with four 1s per row/column + extra 1

Each row wants majority true; each column wants majority false

$$\begin{pmatrix}
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\mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\
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0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & 0 \\
0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 & \color{red}{\mathbf{1}} \\
\mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 & 0 \\
0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 & 0 \\
0 & 0 & \mathbf{1} & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & \mathbf{1} & 0 \\
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Hard for expanding (well spread-out) matrices [MN14]

but easy for regular patterns like the one above (even for DPLL)

Ordering Principle Formulas

"Every finite ordered set $\{e_1, \ldots, e_n\}$ has minimal element"

Variables $x_{i,j} =$ "$e_i < e_j$"

$\overline{x}_{i,j} \lor \overline{x}_{j,i}$	anti-symmetry; not both $e_i < e_j$ and $e_j < e_i$
$\overline{x}_{i,j} \lor \overline{x}_{j,k} \lor x_{i,k}$	transitivity; $e_i < e_j$ and $e_j < e_k$ implies $e_i < e_k$
$\bigvee_{1 \leq i \leq n,\, i \neq j} x_{i,j}$	e_j is not a minimal element

Can also add "total order" axioms

$x_{i,j} \lor x_{j,i}$	totality; either $e_i < e_j$ or $e_j < e_i$

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Conjectured hard [Kri85] but refutable in length $\mathcal{O}(N)$ [Stå96]
Requires resolution width $\Omega(\sqrt[3]{N})$ converted to k-CNF [BG01]
(Or use asymmetric width measure in [Kul99])

Pebbling Formulas

Encode so-called pebble games on DAGs [BW01]

1. $u_1 \oplus u_2$
2. $v_1 \oplus v_2$
3. $w_1 \oplus w_2$
4. $(u_1 \oplus u_2) \wedge (v_1 \oplus v_2) \rightarrow (x_1 \oplus x_2)$
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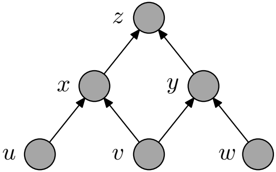
7. $\neg(z_1 \oplus z_2)$



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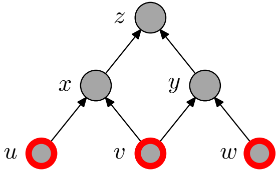

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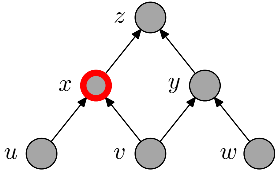

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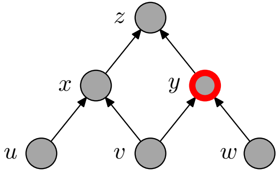

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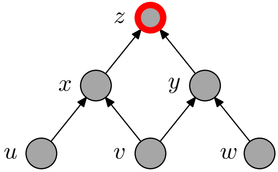

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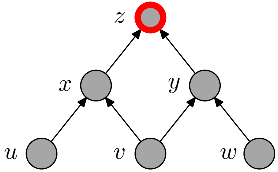

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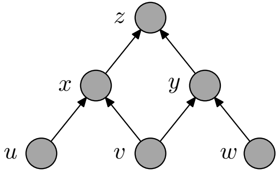

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Write in CNF; e.g., $(x_1 \oplus x_2) \rightarrow (y_1 \oplus y_2)$ becomes

$$
\begin{aligned}
&(x_1 \lor \overline{x}_2 \lor y_1 \lor y_2) \land (x_1 \lor \overline{x}_2 \lor \overline{y}_1 \lor \overline{y}_2) \\
\land\ &(\overline{x}_1 \lor x_2 \lor y_1 \lor y_2) \land (\overline{x}_1 \lor x_2 \lor \overline{y}_1 \lor \overline{y}_2)
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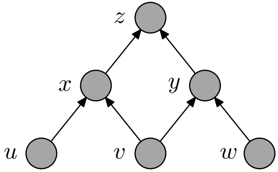

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$$\begin{aligned} &(x_1 \vee \overline{x}_2 \vee y_1 \vee y_2) \wedge (x_1 \vee \overline{x}_2 \vee \overline{y}_1 \vee \overline{y}_2) \\ \wedge\ &(\overline{x}_1 \vee x_2 \vee y_1 \vee y_2) \wedge (\overline{x}_1 \vee x_2 \vee \overline{y}_1 \vee \overline{y}_2) \end{aligned}$$

Pebble game trade-offs $\Rightarrow$ resolution size-space trade-offs [BN08, BN11]

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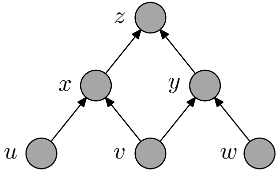

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$$(x_1 \vee \overline{x}_2 \vee y_1 \vee y_2) \wedge (x_1 \vee \overline{x}_2 \vee \overline{y}_1 \vee \overline{y}_2)$$
$$\wedge\ (\overline{x}_1 \vee x_2 \vee y_1 \vee y_2) \wedge (\overline{x}_1 \vee x_2 \vee \overline{y}_1 \vee \overline{y}_2)$$

Pebble game trade-offs $\Rightarrow$ resolution size-space trade-offs [BN08, BN11]
Works for other functions than $\oplus$ (we use NEQ_3, but harder to illustrate)

Instrumented CDCL Solver

To run experiments, add "knobs" to Glucose [AS09, Glu] to analyse:

- restart policy
- branching
- clause database management
- clause learning

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- random shuffling of instances (but doesn't seem to matter)

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- random shuffling of instances (but doesn't seem to matter)

Yields huge number of potential combinations

- Not all combinations make sense, but many do
- Test also settings where “convential wisdom” knows answer
- Several settings still remain to test — marked with $*$ in what follows

CDCL Parameters (1/2)

Restart policy

- No restarts
- LBD-style restarts (Glucose)
- Luby restarts (with different multiplicative factors)*

Variable selection

- Fixed order (chosen to be good)
- VSIDS (with decay factors 0.99^*, 0.95, 0.80, 0.65^*)

Phase saving

- Random phase
- Phase fixed to all false at start of execution*
- Phase fixed randomly at start of execution*
- Standard phase saving

CDCL Parameters (2/2)

Clause erasure

- No clause deletion (keep all learned clauses)
- "Classic" MiniSat-style removal ($\Theta(n)$ clauses after n conflicts)*
- Glucose-style removal ($\Theta(\sqrt{n})$ clauses after n conflicts)
- New, more aggressive MiniSat ($O(n^{0.24})$ clauses after n conflicts)

Clause assessment

- Keep clauses with a good (high) VSIDS score à la MiniSat
- Keep clauses with a good (low) LBD score à la Glucose

Clause learning

- DPLL-style search with minimal amount of clause learning*
- Standard 1UIP clause learning

Some Preliminary Conclusions (1/2)

Importance of restarts

- Sometimes very frequent restarts very important
- Crucial in [AFT11, PD11] for CDCL to simulate resolution efficiently
- Also seems to matter in practice for some formulas which are hard for subsystems of resolution such as regular resolution (stone formulas)

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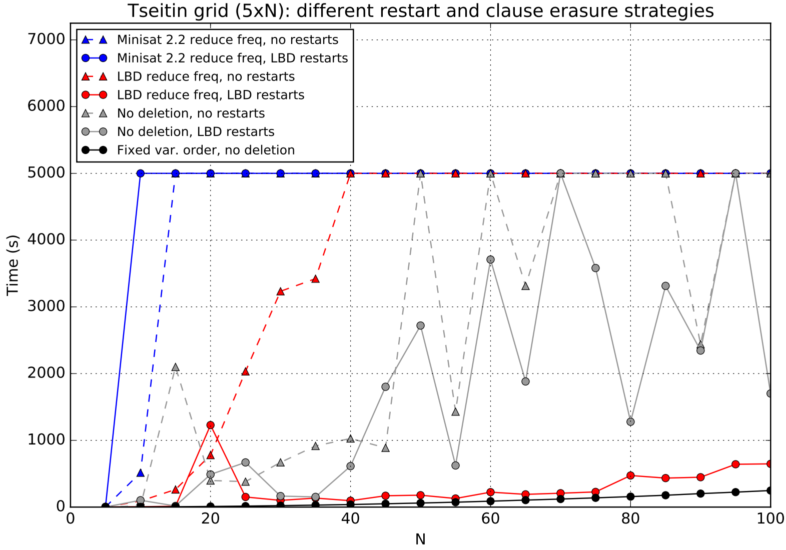
Clause erasure

- Theory says very aggressive clause removal could hurt badly
- Seem to see this on scaled-down versions of time-space trade-off formulas in [BBI12, BNT13] (Tseitin formulas)
- Even no erasure at all can be competitive for these formulas for frequent enough restarts

Plot 1: Tseitin Formulas on Grids



Some Preliminary Conclusions (2/2)

Clause assessment

- Can LBD (literal block distance) heuristic compensate for aggressive erasures by identifying important clauses to keep? Maybe. . .
- But LBD can backfire for too aggressive removal — old glue clauses clog up the clause database(?)

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- Phase saving only helps together with frequent restarts
- Sometimes small variations in VSIDS decay factor (rate of forgetting) absolutely crucial (ordering principle)
- Does slow decay bring solver closer to tree-like resolution???

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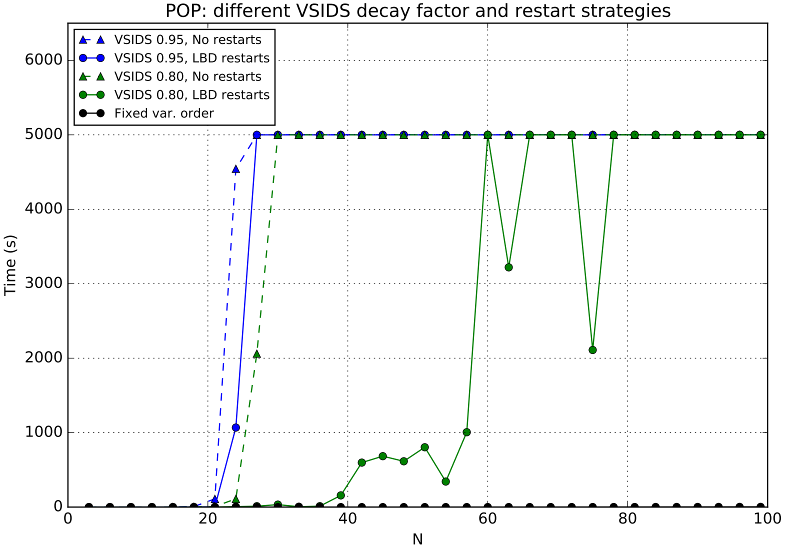
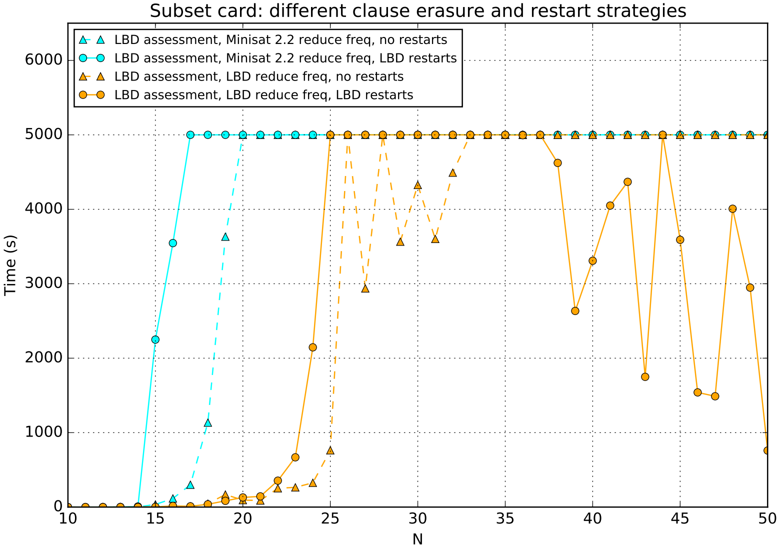
CDCL vs. resolution

- Sometimes CDCL fails miserably on easy formulas (Tseitin, even colouring) — VSIDS just goes dead wrong
- Sometimes strange easy-hard-easy patterns (zero-one designs)

Plot 2: Ordering Principle Formulas



Plot 3: Zero-One Designs



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Thank you for your attention!

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