

# MUSer2: An Efficient MUS Extractor

## SYSTEM DESCRIPTION

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PoS 2012  
June 16, 2012  
Trento, Italy

# Introduction

## Minimal Unsatisfiability

- ▶  $\mathcal{F}$  is *minimally unsatisfiable* ( $\mathcal{F} \in \text{MU}$ ), if  $\mathcal{F} \in \text{UNSAT}$  and for any  $C \in \mathcal{F}$ ,  $\mathcal{F} \setminus \{C\} \in \text{SAT}$ .

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## Example

$$C_1 = x \vee y$$

$$C_3 = x \vee \neg y$$

$$C_2 = \neg x \vee y$$

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- ▶  $\{C_1, C_2, C_3, C_4\} \in \text{MU}$ .

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$$C_3 = x \vee \neg y$$

$$C_5 = y \vee z$$

$$C_2 = \neg x \vee y$$

$$C_4 = \neg x \vee \neg y$$

$$C_6 = y \vee \neg z$$

- ▶  $\{C_1, C_2, C_3, C_4\} \in \text{MU}$ .
- ▶  $\mathcal{F} = \{C_1, \dots, C_6\} \in \text{UNSAT}$ , but  $\notin \text{MU}$ .

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## Applications of MUSes

- ▶ Early 2000's: type debugging in programming languages; circuit error diagnosis; error localization in automotive product configuration data.
- ▶ More recently: model checking (proof-based abstraction refinement); formal equivalence checking; logic synthesis.

# Computation of MUSes

- ▶ Based on detection of *necessary* (or, *transition*) clauses
  - ▶  $C \in \mathcal{F}$  is *necessary* for  $\mathcal{F}$  if  $\mathcal{F} \in \text{UNSAT}$  and  $\mathcal{F} \setminus \{C\} \in \text{SAT}$ .
  - ▶ The set of all necessary clauses of  $\mathcal{F}$  is precisely  $\bigcap \text{MUS}(\mathcal{F})$ .
  - ▶  $\mathcal{F} \in \text{MU}$  if and only if every  $C \in \mathcal{F}$  is necessary for  $\mathcal{F}$ .
  - ▶ If  $C$  is necessary for  $\mathcal{F}$ ,  $C$  is necessary for any  $\text{UNSAT}$  subset of  $\mathcal{F}$ .
- ▶ Iterative calls to SAT solver. Main approaches:
  - ▶ Deletion-based: necessary clauses are detected on transition from  $\text{UNSAT}$  to  $\text{SAT}$ . Unnecessary clauses are removed from the formula. Maintain over-approximation of an MUS.
  - ▶ Insertion-based: necessary clauses are detected on transition from  $\text{SAT}$  to  $\text{UNSAT}$ . Maintain under-approximation of an MUS.
  - ▶ Dichotomic: binary search.
- ▶ SAT solving is the main bottleneck of the computation, hence reduction in the number of SAT solver calls, and making SAT solver calls easier is the key to efficiency.

# MUSer2 features

- ▶ Algorithms:
  - ▶ Hybrid algorithm (default): deletion-based, but builds MUSes bottom-up.
  - ▶ Insertion-based (-ins)
  - ▶ Dichotomic (-dich)
- ▶ Optimizations:
  - ▶ Clause-set refinement (default) and trimming ([`-trim|-tfpl|-tpcrt`])
  - ▶ Recursive model rotation (default)
  - ▶ (Adaptive) redundancy removal ([`-rrl|-rra`])
- ▶ Control/heuristics for clause ordering (`-order`)
- ▶ Testing of computed MUSes (`-test`)
- ▶ SAT solvers are used in a black-box manner; can use various SAT solvers (`-minisat|-picosat`)
- ▶ Software eng.: C++11, designed for extensibility/experimentation.
- ▶ Licensing: source – GPLv3; binaries (incl. extra/experimental features) – free for academic use.

# Hybrid MUS Extraction [Marques-Silva&Lynce'11] w/o optimizations

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

**Output**:  $\mathcal{M} \in \text{MUS}(\mathcal{F})$

```
 $\mathcal{F}' \leftarrow \mathcal{F}$                                 // Working CNF formula
 $\mathcal{M} \leftarrow \emptyset$                             // MUS under-approximation
while  $\mathcal{F}' \neq \emptyset$  do    // Inv:  $\mathcal{M} \subseteq \mathcal{F}$ , and  $\forall C \in \mathcal{M}$  is nec. for  $\mathcal{M} \cup \mathcal{F}'$ 
   $C \leftarrow \text{PickClause}(\mathcal{F}')$ 
  st = SAT( $\mathcal{M} \cup (\mathcal{F}' \setminus \{C\})$ )           // Redundancy removal
  if st = true then          // If SAT,  $C$  is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
     $\mathcal{M} \leftarrow \mathcal{M} \cup \{C\}$ 
    RMR( $\mathcal{F}' \cup \mathcal{M}, \mathcal{M}, \tau$ )           // Recursive model rotation
  else
     $\mathcal{F}' \leftarrow \mathcal{F}' \setminus \{C\}$            // Clause-set refinement
return  $\mathcal{M}$                                 //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
```

- ▶ MUSer2 options: default; [-ins|-dich] to change.

## Optimizations: clause-set refinement/trimming

- ▶ **Fact:** Every unsatisfiable formula contains at least one MUS.
- ▶ Hence, if  $\mathcal{U}$  is an unsatisfiable core of  $\mathcal{F}$ , all clauses outside of  $\mathcal{U}$  can be removed from  $\mathcal{F}$ .
- ▶ Relies on the capability of SAT solvers to return unsatisfiable core.
- ▶ Effect: remove multiple unnecessary clauses at once.
- ▶ Applied to the working formula inside the main loop (e.g.  $\mathcal{M} \cup \mathcal{F}'$  in the Hybrid algorithm) — *clause-set refinement*. Default in MUSer2.
- ▶ Applied to the input formula prior to MUS extraction —  
*clause-set trimming*.
  - ▶ Until fix point: MUSer2 option `-tfp`
  - ▶ A fixed number of times: MUSer2 option `-trim N`
  - ▶ Until size change is bounded: MUSer2 option `-tpcrt P`

# Hybrid MUS Extraction [Marques-Silva&Lynce'11]: clause-set refinement

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

**Output**:  $\mathcal{M} \in \text{MUS}(\mathcal{F})$

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 $\mathcal{F}' \leftarrow \mathcal{F}$                                 // Working CNF formula
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while  $\mathcal{F}' \neq \emptyset$  do    // Inv:  $\mathcal{M} \subseteq \mathcal{F}$ , and  $\forall C \in \mathcal{M}$  is nec. for  $\mathcal{M} \cup \mathcal{F}'$ 
   $C \leftarrow \text{PickClause}(\mathcal{F}')$ 
  st = SAT( $\mathcal{M} \cup (\mathcal{F}' \setminus \{C\})$ )           // Redundancy removal
  if st = true then                         // If SAT,  $C$  is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
     $\mathcal{M} \leftarrow \mathcal{M} \cup \{C\}$ 
    RMR( $\mathcal{F}' \cup \mathcal{M}, \mathcal{M}, \tau$ )          // Recursive model rotation
  else
     $\mathcal{F}' \leftarrow \mathcal{F}' \setminus \{C\}$         // Clause-set refinement
return  $\mathcal{M}$                                 //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
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# Hybrid MUS Extraction [Marques-Silva&Lynce'11]: clause-set refinement

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

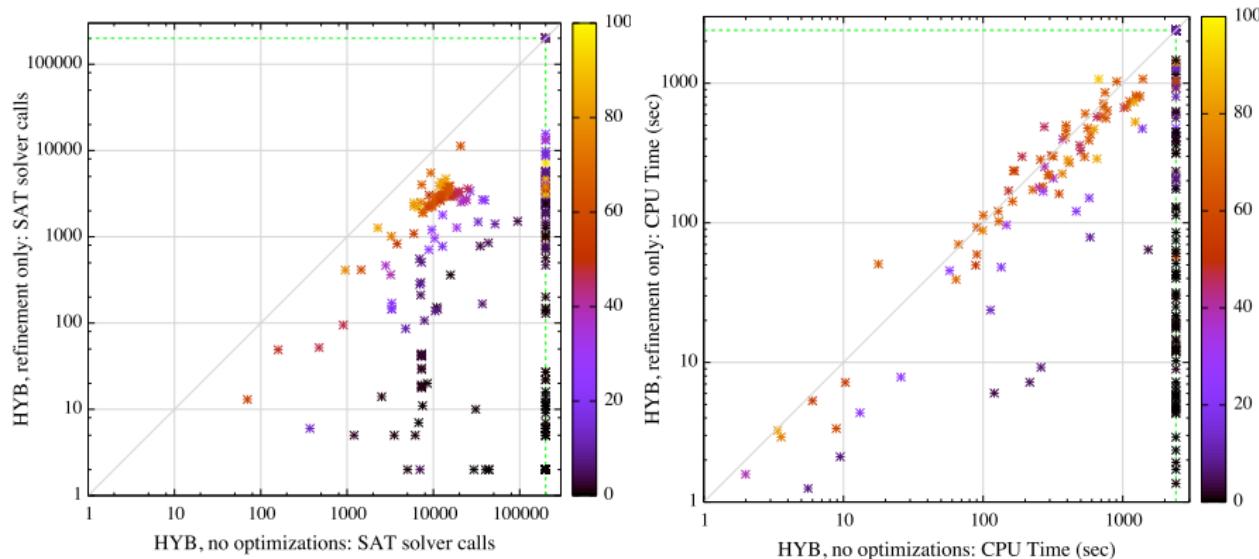
**Output**:  $\mathcal{M} \in \text{MUS}(\mathcal{F})$

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 $\mathcal{F}' \leftarrow \mathcal{F}$                                 // Working CNF formula
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while  $\mathcal{F}' \neq \emptyset$  do    // Inv:  $\mathcal{M} \subseteq \mathcal{F}$ , and  $\forall C \in \mathcal{M}$  is nec. for  $\mathcal{M} \cup \mathcal{F}'$ 
   $C \leftarrow \text{PickClause}(\mathcal{F}')$ 
   $(\text{st}, \mathcal{U}) = \text{SAT}(\mathcal{M} \cup (\mathcal{F}' \setminus \{C\}))$           // Redundancy removal
  if st = true then           // If SAT,  $C$  is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
     $\mathcal{M} \leftarrow \mathcal{M} \cup \{C\}$ 
     $\text{RMR}(\mathcal{F}' \cup \mathcal{M}, \mathcal{M}, \tau)$                 // Recursive model rotation
  else
     $\mathcal{F}' \leftarrow \mathcal{U} \setminus \mathcal{M}$                   // Clause-set refinement
return  $\mathcal{M}$                                 //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
```

- ▶ MUSer2 options: default; `-norf` to disable.

# Impact of clause-set refinement

- ▶ 295 benchmarks from track of SAT Competition 2011.
- ▶ Time limit 1800 sec, memory limit 4 GB.



- ▶ HYB, no optimizations ( $\#sol=132$ ) vs refinement only ( $\#sol=221$ )
  - ▶ Left: number of SAT solver calls. Right: CPU time (sec).
  - ▶ Color: MUS size (% of input size).

# Optimizations: recursive model rotation (RMR)

- ▶ **Fact:**  $C$  is necessary for  $\mathcal{F}$  iff  $\underline{\mathcal{F}} \in \text{UNSAT}$  and  $\exists \tau$  such that  $\text{Unsat}(\mathcal{F}, \tau) = \{C\}$ .  $\tau$  is a **witness** (of necessity) for  $C$ .
  - ▶ During (hybrid) MUS extraction: when  $M \cup (\mathcal{F}' \setminus \{C\}) \in \text{SAT}$ , the assignment  $\tau$  found by the SAT solver is a witness for  $C$ .
  - ▶ Witnesses are also available in other algorithms for MUS extraction.
- ▶ **Model rotation** [Marques-Silva&Lynce'11]: given a witness  $\tau$  for  $C$ , try to modify it into a witness  $\tau'$  for another clause  $C'$ : take  $x \in \text{Var}(C)$ , let  $\tau' = \tau|_{\neg x}$ , if  $\text{Unsat}(\mathcal{F}, \tau') = \{C'\}$ , then  $C'$  is necessary; continue with  $C'$  and  $\tau'$ .
- ▶ **Recursive model rotation** [Belov&Marques-Silva'11]: for each necessary clause explore all possible flips (recursively).
- ▶ Effect: detect multiple necessary clauses in a single SAT solver call.
- ▶ Default in MUSer2.

# Hybrid MUS Extraction [Marques-Silva&Lynce'11]: RMR

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

**Output**:  $\mathcal{M} \in \text{MUS}(\mathcal{F})$

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   $C \leftarrow \text{PickClause}(\mathcal{F}')$ 
   $(\text{st}, \mathcal{U}) = \text{SAT}(\mathcal{M} \cup (\mathcal{F}' \setminus \{C\}))$           // Redundancy removal
  if st = true then           // If SAT,  $C$  is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
     $\mathcal{M} \leftarrow \mathcal{M} \cup \{C\}$ 
     $\text{RMR}(\mathcal{F}' \cup \mathcal{M}, \mathcal{M}, \tau)$                 // Recursive model rotation
  else
     $\mathcal{F}' \leftarrow \mathcal{U} \setminus \mathcal{M}$                   // Clause-set refinement
return  $\mathcal{M}$                                     //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
```

- ▶ MUSer2 options: default; `-norot` to disable.

# Hybrid MUS Extraction [Marques-Silva&Lynce'11]: RMR

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

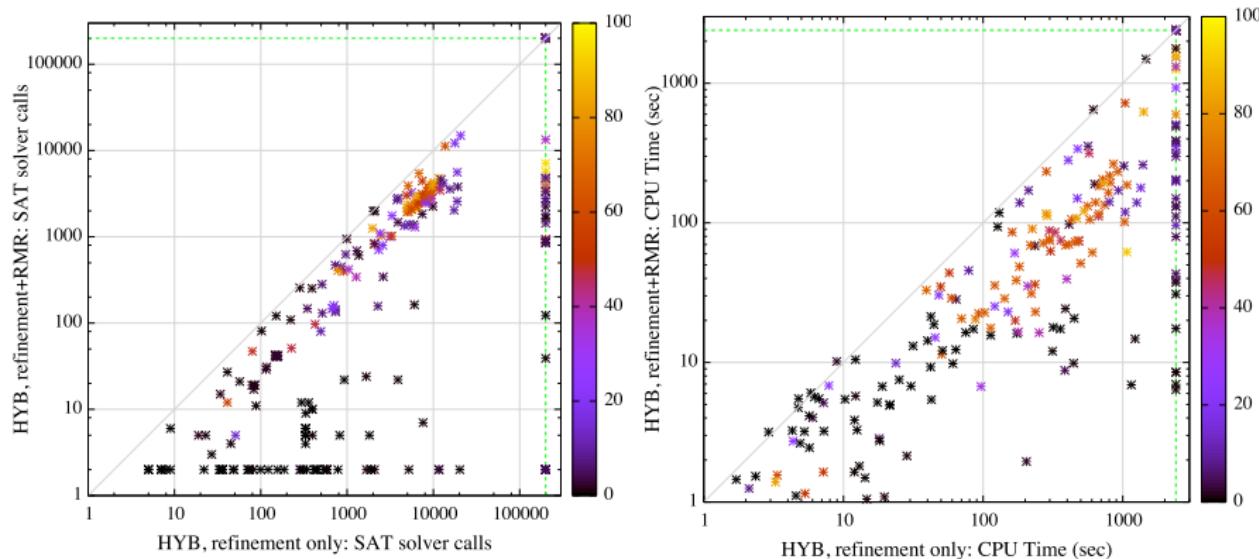
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return  $\mathcal{M}$                                          //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
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- ▶ MUSer2 options: default; `-norot` to disable.

# Impact of recursive model rotation

- ▶ 295 benchmarks from track of SAT Competition 2011.
- ▶ Time limit 1800 sec, memory limit 4 GB.



- ▶ HYB, refinement only (#sol=221) vs refinement+RMR (#sol=254)
  - ▶ Left: number of SAT solver calls. Right: CPU time (sec).
  - ▶ Color: MUS size (% of input size).

# Optimizations: redundancy removal

- ▶ **Fact:** If  $\mathcal{F} \in \text{UNSAT}$ , then  $\mathcal{F} \setminus \{C\} \equiv \mathcal{F} \setminus \{C\} \cup \{\neg C\}$ 
  - ▶  $\{\neg C\}$  stands for  $\bigcup_{l \in C} \neg l$ .
  - ▶ During (hybrid) MUS extraction: add  $\{\neg C\}$  to the formula before SAT solver call [Marques-Silva&Lynce'11].
  - ▶ Can also be done for other algorithms [v.Maaren&Wieringa'08].
- ▶ Effect: make SAT calls easier.
- ▶ But: if  $\mathcal{F} \setminus \{C\} \cup \{\neg C\} \in \text{UNSAT}$  and any of the literals from  $\{\neg C\}$  are in the unsatisfiable core  $\mathcal{U}$ , the core cannot be safely used for refinement ( $\mathcal{F} \cap \mathcal{U}$  may be SAT).
- ▶ Adaptive approach: if a core is “tainted”, disable redundancy removal until the next SAT outcome.
- ▶ MUSer2 options: `-rr|-rra`

# Hybrid MUS Extraction [Marques-Silva&Lynce'11]: redundancy removal

**Input** : Unsatisfiable CNF Formula  $\mathcal{F}$

**Output**:  $\mathcal{M} \in \text{MUS}(\mathcal{F})$

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  if st = true then           // If SAT,  $C$  is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
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return  $\mathcal{M}$                                          //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
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- ▶ MUSer2 options: `-rr`, `-rra` for adaptive.

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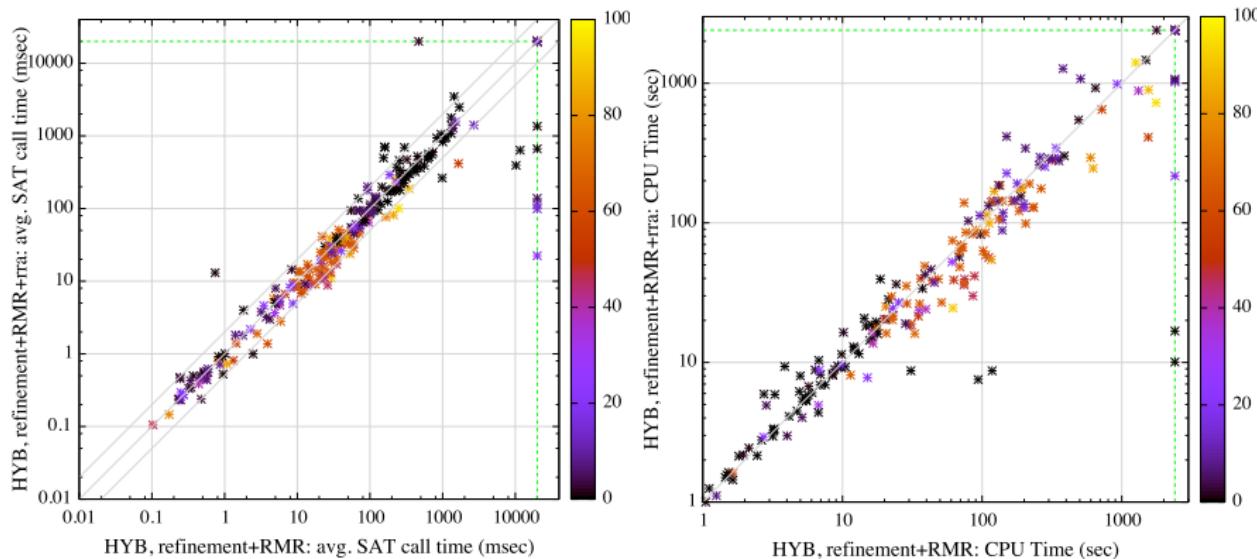
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   $(\text{st}, \tau, \mathcal{U}) = \text{SAT}(\mathcal{M} \cup (\mathcal{F}' \setminus \{C\}) \cup \{\neg C\})$       // Redundancy removal
  if st = true then                      // If SAT, C is necessary for  $\mathcal{M} \cup \mathcal{F}'$ 
     $\mathcal{M} \leftarrow \mathcal{M} \cup \{C\}$ 
    RMR( $\mathcal{F}' \cup \mathcal{M}, \mathcal{M}, \tau$ )          // Recursive model rotation
  else if  $\mathcal{U} \cap \{\neg C\} = \emptyset$  then // If the core is ‘‘clean’’
     $\mathcal{F}' \leftarrow \mathcal{U} \setminus \mathcal{M}$            // Clause-set refinement
return  $\mathcal{M}$                                 //  $\mathcal{M} \in \text{MUS}(\mathcal{F})$ 
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- ▶ MUSer2 options: `-rr`, `-rra` for adaptive.

# Impact of (adaptive) redundancy removal

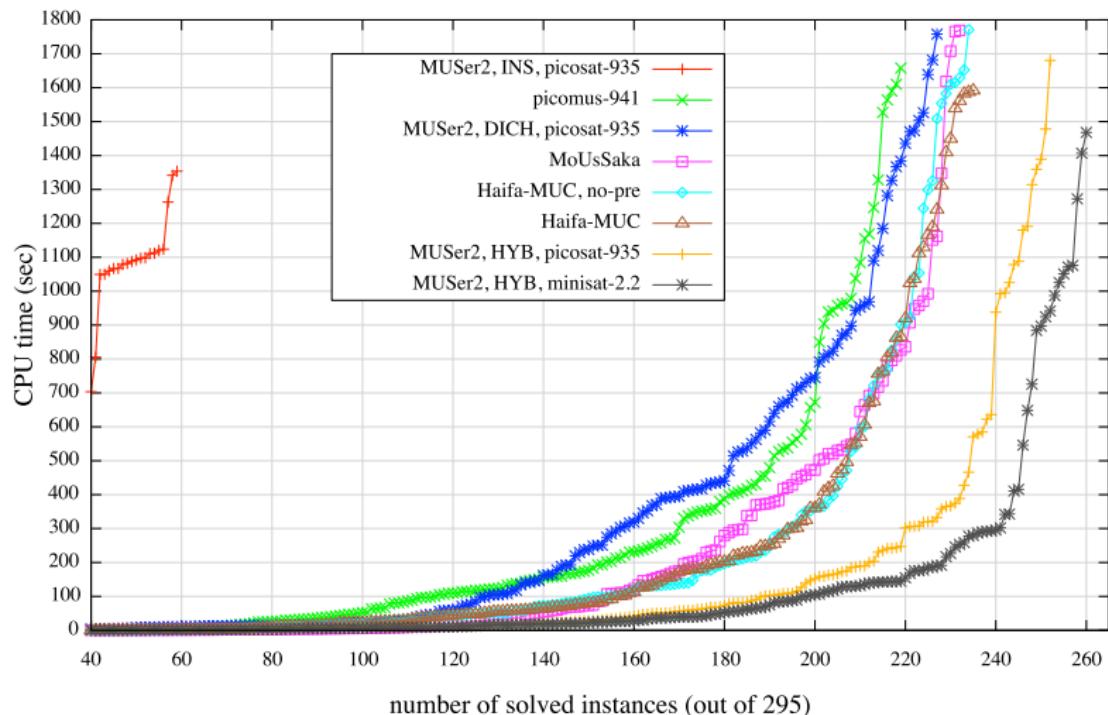
- ▶ 295 benchmarks from track of SAT Competition 2011.
- ▶ Time limit 1800 sec, memory limit 4 GB.



- ▶ HYB, refinement+RMR (#sol=254) vs ref+RMR+rra (#sol=260)
  - ▶ Left: avg. time per SAT call (msec). Right: CPU time (sec).
  - ▶ Color: MUS size (% of input size).

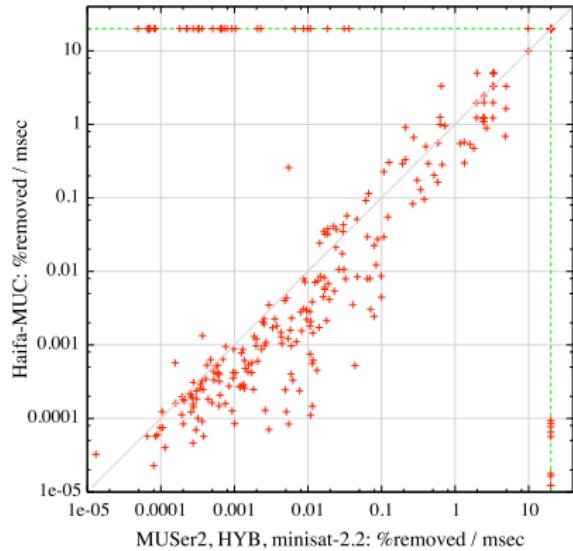
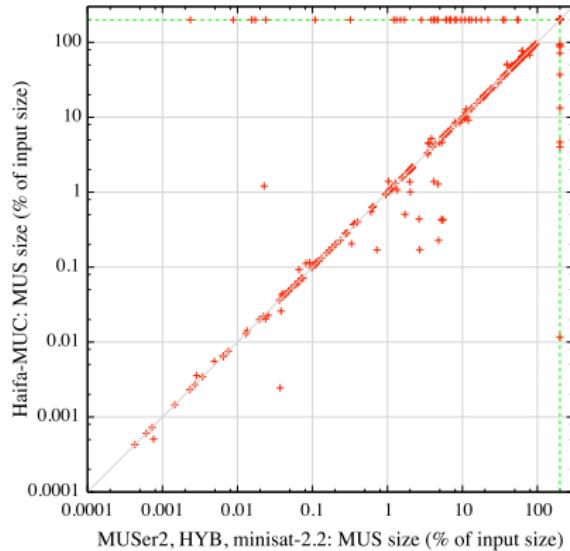
# Performance comparison: run-time

- ▶ 295 benchmarks used in the MUS track of SAT Competition 2011.
- ▶ Time limit 1800 sec, memory limit 4 GB.



# Performance comparison: MUS size and velocity

- ▶ 295 benchmarks from track of SAT Competition 2011.
- ▶ Time limit 1800 sec, memory limit 4 GB.



- ▶ MUSer2 (#sol=260) vs Haifa-MUC (#sol=235)
  - ▶ Left: MUS size (% of input size). Right: velocity (% removed/msec).
  - ▶ Note: the same order.

# Summary

- ▶ MUSer2 — state-of-the-art, open source MUS extractor.
- ▶ Also knows to compute group-MUSes.
  - ▶ All optimizations described in this talk (with the exception of redundancy removal) are implemented for group-MUSes.
- ▶ Single source for all the theory: AI Comm. 2012 [Belov,Lynce&Marques-Silva'12]
- ▶ Binary version: irredundant subformulas [Belov,Janota,Lynce&Marques-Silva'12], variable-MUSes [Belov,Ivrii,Matsliah&Marques-Silva'12], heuristics, and more.
- ▶ TODOs: redundancy removal for group-MUSes/insertion/dichotomic algorithms; wrappers for other SAT solvers.
- ▶ Download at <http://logos.ucd.ie/wiki/doku.php?id=muser>

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